Math 229
Exam 1 - Solutions **Name:**______________________________
01/31/2007

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. Given the points $A : (-1, 4)$ and $B : (5, -2)$:

   (a) (3 points) Find the distance between A and B
   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - (-1))^2 + (-2 - 4)^2} = \sqrt{(6)^2 + (-6)^2}$
   $= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$.

   (b) (3 points) Find the equation for the circle centered at A and passing through the point B.
   Recall that the general equation for a line is: $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle, and r is the radius of the circle.
   Here, since A is the center, $h = -1$ and $k = 4$. Also, since B is on the circle, $r = d(A, b) = 6\sqrt{2}$.
   Therefore, the equation of this circle is: $(x + 1)^2 + (y - 4)^2 = 72$

   (c) (3 points) Find the point C such that B is the midpoint of the line segment connecting A to C
   Let $C = (x, y)$ be the point we are looking for. Then, by the midpoint formula, $(\frac{-1+x}{2}, \frac{4+y}{2}) = B = (5, -2)$. That is, $\frac{-1+x}{2} = 5$, whereby $-1 + x = 10$, so $x = 11$, and $\frac{4+y}{2} = -2$, therefore $4 + y = -4$, so $y = -8$. Thus $C = (11, -8)$.

   (d) (3 points) Find an equation for the line containing A and B
   First, we find the slope: $m = \frac{y_2 - y_1}{x_2 - x_2} = \frac{-2-4}{5+1} = \frac{-6}{6} = -1$.
   Then, by point slope, $y - 4 = -1(x + 1) = -x - 1$, so $y = -x + 3$.

   (e) (3 points) Find an equation for the vertical line containing A
   Recall that vertical lines have fized x-coordinates, and y is allowed to vary.
   Thus the equation for the vertical line containing A is $x = -1$

2. Given the equation $4x + 3y = -2$

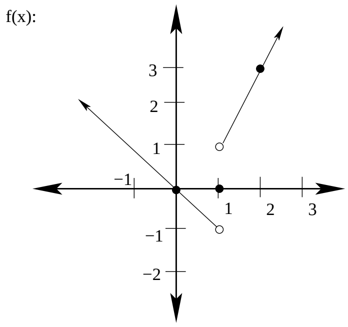
   (a) (3 points) Find the slope of the line represented by this equation.
   Solving for y, we get $3y = -4x - 2$, thus $y = -\frac{4}{3} - \frac{2}{3}$. So the slope is $m = -\frac{4}{3}$

   (b) (4 points) Find the x and y intercepts for this line.
   To find the y-intercept, we set $x = 0$. Then $3y = -2$, or $y = -\frac{2}{3}$. Then the y=intercept is the point $(0, -\frac{2}{3})$.
   To find the x-intercept, we set $y = 0$. Then $4x = -2$, or $x = -\frac{2}{4} = -\frac{1}{2}$. Then the x=intercept is the point $(-\frac{1}{2}, 0)$.

   (c) (3 points) Give an equation for the line that passes through the origin and is parallel to this line.
   Since the lines are parallel, we know that $m = -\frac{4}{3}$. Also, since the line passes through the origin, we know that $b = 0$. Thus the equation is: $y = -\frac{4}{3}x$

3. Given the function
$$f(x) = \begin{cases} -x & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$$

(a) (8 points) Graph $f(x)$.



(b) (6 points) Find the domain and range of $f(x)$. Give your answer in interval notation.

Looking at the graph, we see that no x values are left out, so the domain is: $(-\infty, \infty)$.

Similarly, since all values greater than -1 on the y-axis have points associated with them, the range is: $(-1, \infty)$.

4. Suppose you own a company that manufactures widgets. Your supplier sells you the widgets wholesale at \$8 apiece. It costs you \$750 a month to rent your store, and you spend an additional \$2150 each month on utilities, supplies, and employee salaries. You sell the widgets at a retail price of \$15 apiece.

(a) (3 points) Find an equation $C(x)$ that gives your monthly costs, where x is the number of widgets you purchase for sale that month.

Notice that the fixed costs are $750 + 2150 = 2900$, and the variable costs are \$ per widget. Therefore, $C(x) = 8x + 2900$

(b) (5 points) Find equations for your monthly revenue, $R(x)$, and your monthly profits, $P(x)$, assuming that you sell all of the new widgets that you purchase.

Since we charge \$15 per widget sold, $R(x) = 15x$.

Therefore, $P(x) = R(x) - C(x) = 15x - (8x + 2900) = 7x - 2900$

(c) (4 points) How many widgets do you need to sell each month in order to break even?

To break even, we need $P(x) = 0$, or $7x - 2900 = 0$. Then $7x = 2900$, or $x = \frac{2900}{7} \approx 414.28$. Rounding up, since we can't actually sell .28 widgets, we need to sell at least 415 widgets to avoid losing money.

5. (8 points) Suppose that the supply and demand for a product are given by the equations $2p+3x = 90$ and $4p - 2x = 100$, where x is the quantity sold, in thousands, and p is the price in dollars. Find the equilibrium price for this product, and the quantity sold at this price.

Solution:

Multiplying the first equation by 2 and the second by 3, we get:

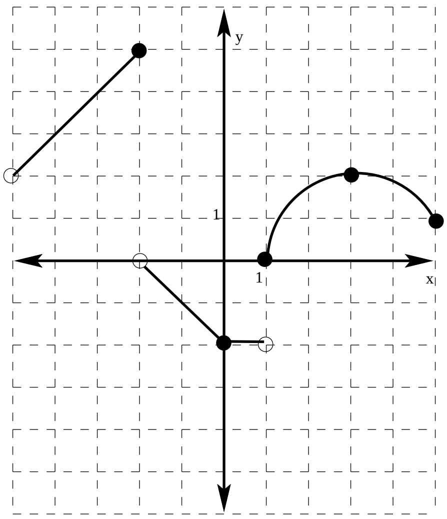
$4p + 6x = 180$, while

$12p - 6x = 300$

$16p = 480$ so $p = 30$.

Plugging this into the first original equation, $2(30) + 3x = 90$, so $3x = 90 - 60 = 30$, thus $x = 10$. Therefore the equilibrium price is \$30, and we would sell 10,000 units at this price.

6. (2 points each) For the given graph of $f(x)$, find the following:



(a) $f(0)$ and $f(3)$

From the graph, $f(0) = -2$ and $f(3) = 2$

(b) x, if $f(x) = 1$

$f(x) = 1$ when $x \approx 1.25$, and when $x = 5$.

(c) The domain of f

Domain: $(-5, 5]$

(d) The range of f

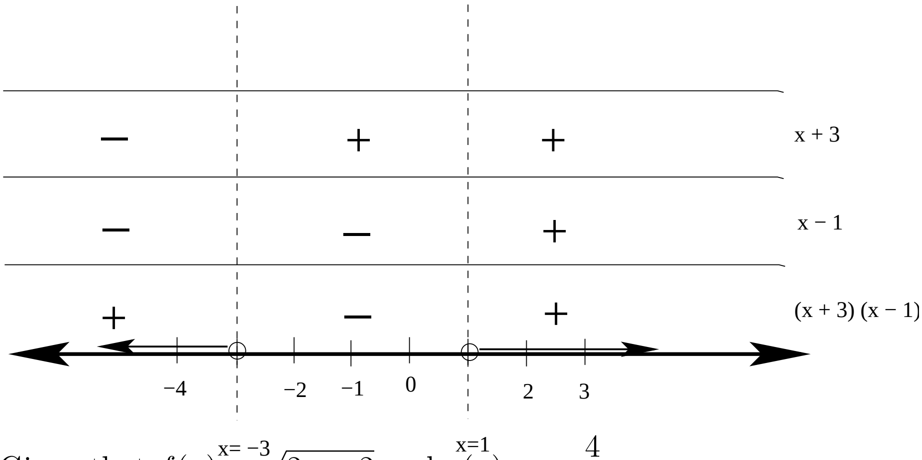
Range: $[-2, 5]$

(e) The intervals where f is increasing.

$(-5, -2] \cup [1, 3]$

7. (8 points) Find the values of x that satifsy the inequality $2x^2 + 3x - 2 > 0$. Graph your solution on a number line.

First notice that this quadratic expression factors to give $(2x-1)(x+2) > 0$. These factors are zero at $x = \frac{1}{2}$ and $x = -2$ respectively. We proceed using sign analysis:



8. Given that $f(x) = \sqrt{2x-2}$ and $g(x) = \dfrac{4}{3x-2}$

(a) (4 points) Find $g(6)$ and $f(3a+1)$
$g(6) = \frac{4}{3(6)-2} = \frac{4}{18-2} = \frac{4}{16} = \frac{1}{4}$
$f(3a+1) = \sqrt{2(3a+1)-2} = \sqrt{6a+2-2} = \sqrt{6a}$

(b) (3 points) Find $\dfrac{g}{f}(3)$
$g(3) = \frac{4}{3(3)-2} = \frac{4}{7}$, and $f(3) = \sqrt{2(3)-2} = \sqrt{6-2} = \sqrt{4} = 2$
Therefore $\frac{g}{f}(3) = \frac{\frac{4}{7}}{2} = \frac{4}{7}\frac{1}{2} = \frac{2}{7}$

(c) (3 points) Find $f \circ g(2)$
$f \circ g(2) = f(g(2)) = f(\frac{4}{3(2)-2}) = f(\frac{4}{4}) = f(1) = \sqrt{2(1)-2} = \sqrt{0} = 0$

9. Given that $f(x) = \sqrt{3x-2}$ and $g(x) = x^2 - 4$

(a) (4 points) Find $g \circ f(x)$
$g \circ f(x) = g(f(x)) = g(\sqrt{3x-2}) = (\sqrt{3x-2})^2 - 4 = (3x-2) - 4 = 3x - 6 = 3(x-2)$

(b) (5 points) Find the domain of $\dfrac{f}{g}$? Give your answer in interval notation.

Notice that $\frac{f}{g}(x) = \frac{\sqrt{3x-2}}{x^2-4}$. Then we must avoid both division by zero and taking the sqare root of a negative number.

First, if $x^2 - 4 = 0$, then $(x-2)(x+2) = 0$, so either $x = 2$ or $x = -2$. Therefore, we need $x \neq \pm 2$.

Next, we must have $3x - 2 \geq 0$, so $3x \geq 2$, or $x \geq \frac{2}{3}$.

Combining these, we see that the domain of $\frac{f}{g}$ is: $[\frac{2}{3}, 2) \cup (2, \infty)$.

(c) (5 points) Find $\dfrac{g(a+h) - g(a)}{h}$. Simplify your answer.
$\frac{g(a+h)-g(a)}{h} = \frac{(a+h)^2-4-(a^2-4)}{h} = \frac{a^2+2ah+h^2-4-a^2+4}{h} = \frac{2ah+h^2}{h} = \frac{h(2a+h)}{h} = 2a + h.$