

**Instructions:** You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. Evaluate the following limits. Be sure to show enough work to justify your answers.

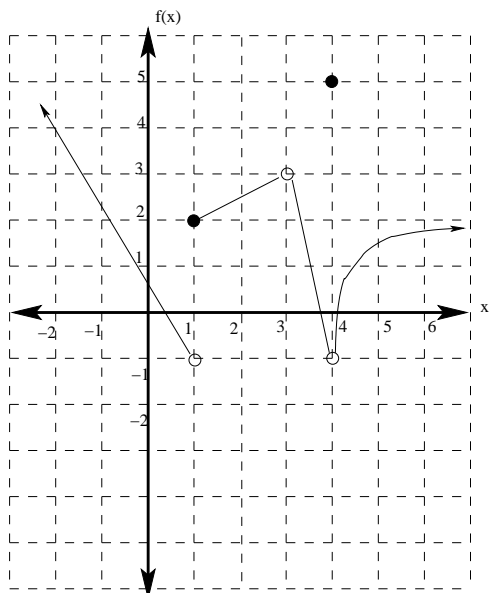
(a) (4 points)  $\lim_{x \rightarrow 0} \frac{x^2 - 2x}{2x^2 - x - 6}$

(b) (4 points)  $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{2x^2 - x - 6}$

(c) (4 points)  $\lim_{x \rightarrow 2} \frac{x}{x - 2}$

(d) (4 points)  $\lim_{x \rightarrow \infty} \frac{x^2 - 2x}{2x^2 - x - 6}$

2. Given the following graph:



(a) (3 points) Find  $\lim_{x \rightarrow 1^-} f(x)$

(b) (3 points) Find  $\lim_{x \rightarrow 1^+} f(x)$

(c) (3 points) Find  $\lim_{x \rightarrow 4} f(x)$

(d) (3 points) Find  $\lim_{x \rightarrow \infty} f(x)$

(e) (5 points) List all points where  $f(x)$  is discontinuous. Explain what goes wrong at each point.

3. (10 points) Use the limit definition of the derivative to compute the derivative function  $f'(x)$  if  $f(x) = 4 - 2x - 3x^2$

4. Suppose  $f(x) = x^3 - 3x^2 + 5$ .

(a) (5 points) Find the equation for the tangent line to  $f(x)$  when  $x = 1$ .

(b) (5 points) Find the value(s) of  $x$  for which the tangent line to  $f(x)$  is horizontal.

5. Find the derivative of each of the following functions. You **do not** have to use the limit definition, and you **do not** need to simplify your answers.

(a) (6 points)  $h(x) = x^3 + \sqrt{x^3}$

(b) (6 points)  $h(x) = \frac{5x^3 - 4x^2 + 7x}{x^2}$

(c) (6 points)  $h(x) = (x^2 - 4x^3)(4x^3 + 3x^2 - 7x + 3)$

(d) (6 points)  $h(x) = (x^3 - 2x + 1)^{\frac{5}{2}}$

(e) (6 points)  $\left(\frac{2 - 4x^3}{x^2 - 1}\right)^4$

6. Suppose you own a company that manufactures widgets, and the demand equation for them is given by  $3x + 4p = 120$ .

(a) (5 points) Find the revenue function  $R(x)$ , and use it to compute  $R(10)$  and  $R(40)$ .

(b) (4 points) Find the marginal revenue function  $R'(x)$

(c) (4 points) Compute  $R'(10)$  and  $R'(40)$  and explain what these numbers mean in practical terms.

(d) (5 points) If  $C(x) = 20x + \frac{1}{4}x^2 + 100$ , find  $P(x)$  and use it to compute  $P(10)$ .

(e) (4 points) Find the marginal profit function  $P'(x)$ , use it to compute  $P'(5)$ , and explain what this means in practical terms.