Math 229
Exam 3
03/09/2007

Name:_____

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. (7 points each)

   (a) Find the third derivative of $f(x)$, given that $f(x) = \dfrac{3}{x^2}$

   (b) Find the second derivative of $g(x)$, given that $g(x) = \dfrac{2x+1}{3x+2}$

   (c) Find the second derivative of $g(x)$, given that $h(x) = (1-x^2)^7$

2. (5 points each) Determine whether the following statements are True or False. Write a brief explanation to justify your answer.

   (a) If $f'(a) = 0$ and $f''(a) < 0$, then $(a, f(a))$ is a relative minimum of the function $f(x)$.

   (b) If $f'(x) > 0$ for $a \leq x \leq b$, then $(a, f(a))$ is an absolute minimum for $f(x)$ on $[a, b]$.

3. (6 points each) Let $f(x) = x^4 - 8x^3 + 16x^2$

   (a) Find the x and y intercepts of $f(x)$.

   (b) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

   (c) Find and classify the relative extrema of $f(x)$.

   (d) Find the equation of the tangent line to $f(x)$ when $x = 1$.

4. Given that $f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3$ and $f'(x) = x^4 - 2x^2$:

   (a) (8 points) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

   (b) (6 points) Find the coordinates of the inflection points of $f(x)$.

5. (15 points) Carefully draw the graph of a function satisfying the following conditions:
   x-intercepts: $(-2, 0), (2, 0), (5, 0)$; y-intercept: $(0, 5)$
   Increasing on $(-\infty, 0) \cup (3, \infty)$ and Decreasing on $(0, 3)$
   Concave Up on $(-1, 0) \cup (2, \infty)$ and Concave Down on $(-\infty, -1) \cup (0, 2)$
   $f(-1) = 3$, and $f(3) = -4$.

6. Suppose the daily cost for producing x widgets is given by $C(x) = 5x^2 - 20x + 500$, where $C(x)$ is in dollars, and a maximum of 20 widgets can be produced each day.

   (a) (8 points) Find the production level which minimizes the daily costs. Also find the daily cost at this production level.

   (b) (8 points) Find the production level which minimizes the **average** cost per widget. Also find the average cost per sidget at this production level.