

Instructions: You will have 50 minutes to complete this exam. Calculators are allowed, but this is a closed book, closed notes exam. I will give credit to each problem proportional to the amount of correct work shown. Correct answers without supporting work will receive little credit. Be sure to simplify answers when possible. Also, make sure to follow directions carefully on each problem.

1. (2 points each) Determine whether the following are True or False:

(a) $\ln\left(\frac{x^3}{(x+1)(x-1)}\right) = 3\ln x - \ln(x+1) + \ln(x-1)$

(b) $e^{\ln(x^2+1)} = x^2 + 1$

(c) $e^{x^2} \cdot e^{3x} = e^{3x^3}$

(d) $\frac{\ln(4x)}{\ln(2x)} = \ln 2$

(e) $\ln(e^{x^2} - 4) = x^2 - \ln 4$

2. Suppose that a culture of bacteria that initially has 500 cells grows to 10,000 cells in 12 hours.

(a) (5 points) Find a function $f(t)$ that gives the number of cells in the culture as a function of time (in hours), assuming that this population grows continuously and exponentially.

(b) (5 points) How long will it take for the culture to reach 1,000,000 cells?

3. (7 points) Find the interest rate needed for an investment of \$1,000 to double in 7 years if the interest is compounded monthly.

4. (6 points each) Compute the derivatives of the following functions. You do not need to simplify your answers.

(a) $f(x) = \ln\left(\frac{x^2}{(2x-1)^3}\right)$

(b) $g(x) = e^{e^{2x^3}}$

(c) $h(x) = \ln(x^2 + 1)e^{x^3}$

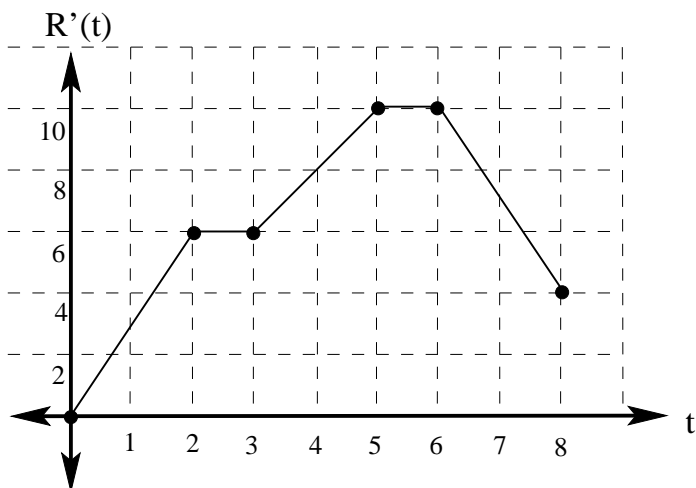
(d) $l(x) = (3x - 2)^4(x^2 - 3x)^{\frac{5}{2}}(x^3 - 1)^{\frac{4}{3}}$

5. (7 points) Find the tangent line to $f(x) = 2xe^{2x-4}$ when $x = 2$

6. (8 points) Determine the intervals where the function $g(t) = t^4e^{2t}$ is increasing and the intervals where it is decreasing.

7. (4 points) Suppose $\int_0^2 f(x) dx = 4$ and $\int_0^2 g(x) dx = 2$. Find $\int_0^2 2f(x) - g(x) dx$

8. (8 points) Suppose marginal revenue, $R'(t)$ is given by the graph below, where t is in months and $R'(t)$ is in \$1000s of dollars per month. Find the total revenue from $t = 0$ to $t = 6$.



9. (6 points each) Evaluate the following integrals:

(a) $\int 5x^4 - x^{\frac{3}{2}} dx$

(b) $\int \frac{4x^2 + 2}{x^3} dx$

(c) $\int_0^3 4x^2 - e^{3x} dx$

10. (8 points) Find the area of the region enclosed by the graphs $f(x) = x$ and $g(x) = \sqrt{x}$.