

Antiderivatives and Integration

A. Antiderivatives

A function F is an **antiderivative** of f if $F'(x) = f(x)$.

Example: $F(x) = x^3$ is an antiderivative of $f(x) = 3x^2$, since $F'(x) = 3x^2 = f(x)$.

Note: If $F(x)$ is an antiderivative of f , then so is $F(x) + C$ for any constant C .

B. Indefinite Integrals

The notation $\int f(x) dx = F(x) + C$ is used to denote the indefinite integral of the function $f(x)$ with respect to the variable x . Here, $F(x) + C$ is the general antiderivative of $f(x)$.

C. Integration Rules

1. Integrating a constant: $\int a dx = ax + C$ for any constant a .

Example: $\int \frac{3}{2} dx = \frac{3}{2}x + C$

2. Integrating Power Functions: $\int x^r dx = \frac{1}{r+1}x^{r+1} + C$

Example: $\int x^{\frac{3}{2}} dx = \frac{2}{5}x^{\frac{5}{2}} + C$

3. Constant Multiples: $\int af(x) dx = a \int f(x) dx$ for any constant a .

Example: $\int 7x^{\frac{3}{2}} dx = 7 \int x^{\frac{3}{2}} dx = (7)\frac{2}{5}x^{\frac{5}{2}} + C = \frac{14}{5}x^{\frac{5}{2}} + C$

4. Sums and Differences: $\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$,

and $\int f(x) - g(x) dx = \int f(x) dx - \int g(x) dx$ for any functions f and g .

Example: $\int 3x^2 + x^{\frac{3}{2}} dx = \int 3x^2 dx + \int x^{\frac{3}{2}} dx = x^3 + \frac{2}{5}x^{\frac{5}{2}} + C$

5. Integrals Resulting in Exponentials and Natural Logarithms:

$$\int e^x dx = e^x + C, \text{ and } \int \frac{1}{x} dx = \ln |x| + C.$$

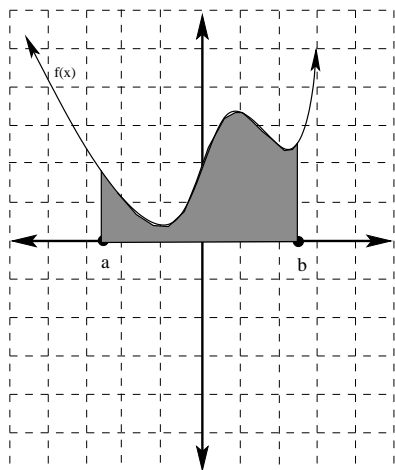
D. Definite Integrals

The Fundamental Theorem of Calculus:

Let f be a continuous function on the interval $[a, b]$, and let F be an antiderivative of f .

Then $\int_a^b f(x) dx = F(b) - F(a)$

Note: The number resulting from this computation is the area "under" the graph of $f(x)$ between $x = a$ and $x = b$.



Example: $\int_1^3 3x^2 dx = x^3 \Big|_1^3 = 3^3 - 1^3 = 27 - 1 = 26$

E. Properties of Definite Integrals

1. $\int_a^a f(x) dx = 0$

2. $\int_a^b f(x) dx = - \int_b^a f(x) dx$

3. $\int_a^b cf(x) dx = c \int_a^b f(x) dx$, for any constant c

4. $\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

5. $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$, for any $a < c < b$

F. The Average Value of a Function

Let f be a function that is integrable on an interval $[a, b]$. Then the **average value** of f over $[a, b]$ is

$$\frac{1}{b-a} \int_a^b f(x) dx.$$