Math 229
Practice Exam 2

1. Evaluate the following limits:

   (a) $\lim\limits_{x \to 0} \dfrac{2x^2 - x - 1}{x^2 - 1}$

   (b) $\lim\limits_{x \to 1} \dfrac{2x^2 - x - 1}{x^2 - 1}$

   (c) $\lim\limits_{x \to \infty} \dfrac{2x^2 - x - 1}{x^2 - 1}$

2. Given the function
$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$$

   (a) Graph $f(x)$.

   (b) Find $\lim\limits_{x \to 1} f(x)$.

   (c) Is $f(x)$ continuous at $x = 1$? Justify your answer.

3. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 3x^2 - 2x + 1$

4. Suppose $f(x) = (x + 1)^{\frac{3}{2}}$.

   (a) Find the equation for the tangent line to $f(x)$ when $x = 3$.

   (b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

5. Find the derivative of each of the following functions. You **do not** have to use the limit definition, and you **do not** need to simplify your answers.

   (a) $f(x) = 5x^4 - 3x^2 + \frac{2}{x}$

   (b) $f(x) = (2x^2 + 5x - 4)(x^3 + 2x^2 - 1)$

   (c) $f(x) = \dfrac{2x + 3}{x^2 - 1}$

   (d) $f(x) = \sqrt{2x^2 + 1}$

   (e) $f(x) = (x^2 + 1)(x^3 - 2x + 1)^{\frac{3}{2}}$

6. Suppose you own a company that manufactures widgets, and the demand equation for them is given by $5x + 4p = 200$.

   (a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(30)$.

   (b) Find the marginal revenue function $R'(x)$

   (c) Compute $R'(10)$ and $R'(30)$ and explain what these numbers mean in practical terms.

   (d) If $C(x) = -x^2 + 25x + 100$, find $P(x)$, the profit function, and $P'(x)$, the marginal profit function.

   (e) Compute $P(10)$ and $P'(10)$ and explain what these numbers mean in practical terms.