Math 229
Quiz 3 - Solutions **Name:**____________________________
Due: 03/05/2007

This is a Take-Home Quiz. You may use your book and course notes, and you may consult with other members of the class, but you may not consult with outside tutors (at least not on these specific problems).

1. (3 points) Given $f(x) = (3x^2 - 2x + 5)^{\frac{7}{3}}$, find $f''(x)$.

   $f'(x) = \frac{7}{3}(3x^2 - 2x + 5)^{\frac{4}{3}}(6x - 2) = (14x - \frac{14}{3})(3x^2 - 2x + 5)^{\frac{4}{3}}$

   $f''(x) = 14(3x^2 - 2x + 5)^{\frac{4}{3}} + (14x - \frac{14}{3})(\frac{4}{3})(3x^2 - 2x + 5)^{\frac{1}{3}}(6x - 2)$

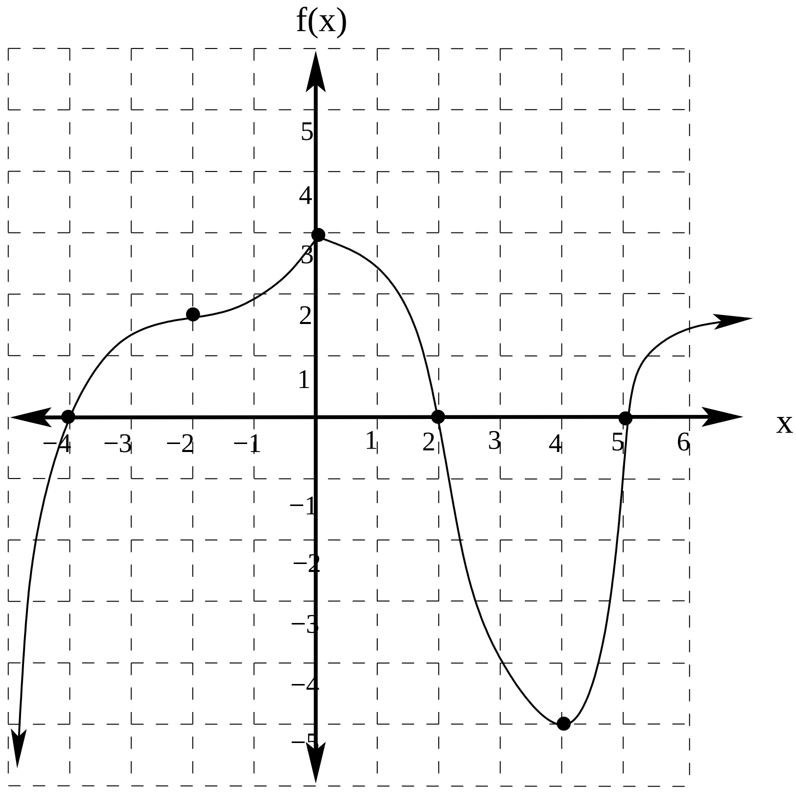
2. (7 points) In the space below, sketch the graph of a function satisfying the following properties:

   x-intercepts: $(-4, 0), (2, 0), (5, 0)$

   Increasing on: $(-\infty, 0) \cup (4, \infty)$, Decreasing on: $(0, 4)$

   Concave up on: $(-2, 0) \cup (2, 5)$, Concave down on: $(-\infty, -2) \cup (0, 2) \cup (5, \infty)$

   Local Max: $(0, 3)$; Local Min: $(4, -5)$.



3. (10 points) Suppose $f(x) = x^4 - 24x^2$.

(a) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

(b) Find all relative maxima and relative minima of $f(x)$.

(c) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.

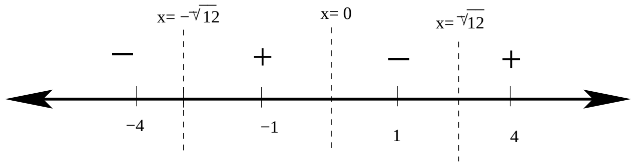
(d) Find any inflection points of $f(x)$.

(e) Sketch the graph of $f(x)$ accurately enough to show all relative extrema and inflection points.

(a) To find increasing/decreasing behavior of $f(x)$, we look at the first derivative:

$f'(x) = 4x^3 - 48x = 4x(x^2 - 12)$. Then the critical points occur when $4x(x^2 - 12) = 0$, that is, when $x = 0$, or when $x^2 = 12$, that is $x = \pm\sqrt{12}$.

Sign testing, we see:



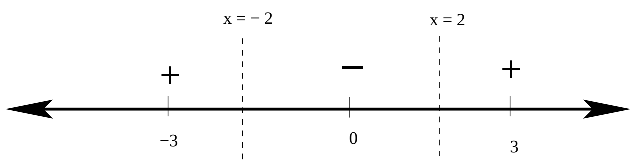
Therefore, $f(x)$ is increasing on $(-\sqrt{12}, 0) \cup (\sqrt{12}, \infty)$, and $f(x)$ is decreasing on $(-\infty, -\sqrt{12}) \cup (0, \sqrt{12})$

(b) From the sign diagram above, we see that $f(x)$ has a relative maximum when $x = 0$, that is, at $(0, f(0)) = (0, 0)$, and relative minima when $x = \pm\sqrt{12}$, that is, at $(-\sqrt{12}, f(-\sqrt{12})) = (-\sqrt{12}, -144)$ and at $(\sqrt{12}, f(\sqrt{12})) = (\sqrt{12}, 144)$.

(c) To understand the concavity of $f(x)$, we look at the second derivative:

$f''(x) = 12x^2 - 48$, which has key values when $12x^2 - 48 = 0$, that is, when $12x^2 = 48$, $x^2 = 4$, or $x = \pm 2$.

Sign testing, we see:



Therefore, $f(x)$ is concave up on $(\infty, -2) \cup (2, \infty)$, and $f(x)$ is concave down on $(-2, 2)$

(d) From the sign diagram above, we see that $f(x)$ has inflection points when $x = \pm$, that is, at $(-2, f(-2)) = (2, -80)$, and at $(2, f(2)) = (2, -80)$.

(e) Putting all this information together: