

This is a Take-Home Quiz. You may use your book and course notes, and you may consult with other members of the class, but you may not consult with outside tutors (at least not on these specific problems). You may use the back of this paper if you run out of room on the front.

1. (10 points) Solve the following system of linear equations using matrix methods:

$$\begin{cases} 3x - 5z = 8 \\ 2x - y + 2z = -2 \\ 4x + 3y = 10 \end{cases}$$

We will solve this system by changing to matrix form and transforming the matrix form of this system:

$$\begin{aligned} & \left[\begin{array}{ccc|c} 3 & 0 & -5 & 8 \\ 2 & -1 & 2 & -2 \\ 4 & 3 & 0 & 10 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & -7 & 10 \\ 2 & -1 & 2 & -2 \\ 4 & 3 & 0 & 10 \end{array} \right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & -7 & 10 \\ 0 & -3 & 16 & -22 \\ 4 & 3 & 0 & 10 \end{array} \right] \\ & \xrightarrow{R_3 - 4R_1} \left[\begin{array}{ccc|c} 1 & 1 & -7 & 10 \\ 0 & -3 & 16 & -22 \\ 0 & -1 & 28 & -30 \end{array} \right] \xrightarrow{-R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 1 & -7 & 10 \\ 0 & -1 & 28 & -30 \\ 0 & -3 & 16 & -22 \end{array} \right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 21 & -20 \\ 0 & -1 & 28 & -30 \\ 0 & -3 & 16 & -22 \end{array} \right] \\ & \xrightarrow{R_3 + 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 21 & -20 \\ 0 & -1 & 28 & -30 \\ 0 & 0 & -68 & 68 \end{array} \right] \xrightarrow{-\frac{1}{68}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 21 & -20 \\ 0 & -1 & 28 & -30 \\ 0 & 0 & 1 & -1 \end{array} \right] \xrightarrow{R_1 - 21R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 28 & -30 \\ 0 & 0 & 1 & -1 \end{array} \right] \\ & \xrightarrow{R_2 + 28R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{array} \right] \end{aligned}$$

Therefore, the solution to this system is $x = 1$, $y = 2$, and $z = -1$

Check: Plugging these values back into our original system, we obtain

$$\begin{cases} 3(1) - 5(-1) = 3 + 5 = 8 \\ 2(1) - 2 + 2(-1) = 2 - 2 - 2 = -2 \\ 4(1) + 3(2) = 4 + 6 = 10 \end{cases}$$