

You MUST use good notation
and show appropriate work.

Math 102
(Section 2.5B Supplemental)

Name _____

2.5B Supplemental

As arguments become more complicated it is often impractical to check an argument for validity by using truth tables. Instead of using truth tables to test whether a conclusion is valid, mathematicians often use a formal proof. We will restrict our formal proofs to those that are called direct proofs. By a direct proof we will mean a sequence of true statements such that:

- i)* the statement is a premise which is given to be true, or
- ii)* the statement is the conclusion of a valid argument from a given list of valid arguments such that its premises are from preceding statements in the sequence or
- iii)* the statement is a logically equivalent statement to a preceding statement in the sequence, and such that
- iv)* the last statement in the sequence is the statement (conclusion) to be proved.

Some useful valid arguments (forms)

- 1) Law of Detachment $p \rightarrow q, p, \therefore q$
- 2) Law of Contraposition $p \rightarrow q, \sim q, \therefore \sim p$
- 3) Law of Syllogism $p \rightarrow q, q \rightarrow r, \therefore p \rightarrow r$
- 4) Disjunctive Syllogism $p \vee q, \sim p, \therefore q$
- 5) Simplification $p \wedge q, \therefore p$ (or could conclude q)
- 6) Addition $p, \therefore p \vee q$

Some logically equivalent statements (forms)

- 1) Double negation p and $\sim(\sim p)$
- 2) Contraposition $p \rightarrow q$ and $\sim q \rightarrow \sim p$
- 3) De Morgan's laws
 - i)* $\sim(p \wedge q)$ and $\sim p \vee \sim q$
 - ii)* $\sim(p \vee q)$ and $\sim p \wedge \sim q$
- 4) Conditionals to Disjunctions $p \rightarrow q$ and $\sim p \vee q$
- 5) Commutative $p \wedge q$ and $q \wedge p$, also $p \vee q$ and $q \vee p$

(over)

1. Examples of formal proofs.

a) Construct a “formal proof” in order to establish that the following argument is valid:

$$\begin{array}{l} (q \vee r) \rightarrow p \\ \sim p \\ \hline s \rightarrow r \\ \therefore \sim s \end{array}$$

Solution:

<u>Statement</u>	<u>Reason</u>
1. $(q \vee r) \rightarrow p$	premise
2. $\sim p$	premise
3. $s \rightarrow r$	premise
4. $\sim (q \vee r)$	1, 2 law of contraposition
5. $\sim q \wedge \sim r$	4, De Morgan
6. $\sim r$	5 simplication
7. $\therefore \sim s$	3, 6 law of contraposition

b) Following is a possible list of statements in a formal proof of the argument form:

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline p \vee s \\ \therefore s \end{array}$$

Provide reasons for each statement.

<u>Statement</u>	<u>Reason</u>
1. $p \rightarrow q$	
2. $p \vee s$	
3. $\sim p \rightarrow s$	
4. $\sim q \rightarrow \sim p$	
5. $\sim q \rightarrow s$	
6. $\sim q$	
7. $\therefore s$	

2. Some practice examples

a)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \sim q \\ \hline \therefore p \rightarrow \sim r \end{array}$$

b)
$$\begin{array}{l} \sim s \\ p \rightarrow r \\ \hline (p \rightarrow q) \rightarrow s \\ \therefore r \end{array}$$

c)
$$\begin{array}{l} \sim p \rightarrow (q \vee p) \\ \hline \sim p \\ \therefore q \end{array}$$