Math 102
Exam 2: Additional Practice Problems

1. Negate each of the following statements, then rewrite them as English sentences:
   (a) All bees are busy.
   (b) Some things are better left unsaid.
   (c) I got up early on Saturday and went to the gym.
   (d) This summer I will get a job or I will take classes.
   (e) If I eat my vegetables then I will get dessert

2. Given p : I studied for this exam, q : I got a good grade on this exam, r : I understand truth tables, and s : I am not good at doing proofs, translate the following statements into words:
   (a) $p \wedge (\sim s) \rightarrow q$
   (b) $(\sim p \vee s) \rightarrow \sim q$
   (c) $(p \rightarrow (r \wedge (\sim s)) \rightarrow q$

3. (a) Explain, in your own words, the difference between "exclusive or" and "inclusive or"
   (b) Give real world examples that illustrate both "exclusive or" and "inclusive or"

4. Given the statements: p : There is a full moon tonight, and q : I will go for a walk on the beach
   (a) Write the conditional statement relating p to q in words.
   (b) Write the converse in words.
   (c) Write the inverse in words.
   (d) Write the contrapositive in words.
   (e) Indicate which of these statements above are logically equivalent to each other. You do not need to prove your answer.

5. According to one of DeMorgan's Laws, $\sim (p \vee q)$ is logically equivalent to $(\sim p) \wedge (\sim q)$. Use truth tables to prove that these two statements are logically equivalent. Then, explain in your own words why the fact that these two statements are equivalent makes sense.

6. Given that p is true, q is false, r is true, and s is true:
   (a) What is truth value of the statement: $\sim (p \vee q) \rightarrow (r \wedge \sim s)$
   (b) How many rows would the full truth table for the expression $\sim (p \vee q) \rightarrow (r \wedge \sim s)$ have?

7. Build truth tables for the following logical statements:
   (a) $(p \wedge (\sim q) \rightarrow q$
   (b) $\sim q \rightarrow (p \vee \sim r)$
   (c) $(p \rightarrow q) \leftrightarrow \sim (q \wedge r)$

8. Identify the form of the following arguments, and state whether the given argument is valid:
   (a) If I have enough money saved up, then I will go to Mexico for Spring Break. I did not go to Mexico for Spring Break. Therefore, I did not have enough money saved up.
   (b) If I lie on my tax return, then I will get audited by the IRS. I got audited by the IRS. Therefore, I lied on my tax return.
   (c) I will go to Mexico for Spring Break or I will spend Spring Break with my family. I did not spend Spring Break with my family. Therefore, I went to Mexico for Spring Break.

9. (a) Draw an Euler diagram for the statements: "Some A's are B's", "All C's are not A's", and " All D's are A's"

   (b) State a valid conclusion that can be made based on the statements in part (a) above.

10. Use Euler diagrams to determine whether the following syllogisms are valid or invalid:

    (a)
    Some exams are too long.
    Some exams are too difficult.

    Therefore, some exams are too long and too difficult.

    (b)
    Some dogs chase cats.
    All dogs have fleas.

    Therefore, some cat-chasing dogs have fleas.

11. Use a truth table to determine whether or not the following argument is valid:

    If I work hard, then I will get a raise.
    If I get a raise, then I will not have to get a second job.
    I got a second job.

    Therefore, I did not work hard.

12. Given the argument:

    $p \rightarrow q$
    $\sim (q \wedge r)$
    r

    $\therefore \sim p$

    Fill in the missing reasons in the following two column proof:

Statement	**Reason**
1. $\sim (q \wedge r)$	
2. $\sim q \vee \sim r$	
3. r	
4. $\sim (\sim r)$	
5. $\sim q$	
6. $p \rightarrow q$	
7. $\sim q \rightarrow \sim p$	
8. $\sim p$	

13. Write a 2-column proof to verify the following argument:

    $t \rightarrow p$
    $s \vee t$
    $p \rightarrow q$
    $\sim q$

    $\therefore s$

14. Use basic counting principles to find each of the following:

    (a) Suppose I have 5 shirts, 4 pairs of pants, and 3 pairs of shoes. How many different outfits could I wear?

    (b) Suppose I first flip a coin and then roll two 6-sided dice. How many distinct outcomes are there if I only care which side of the coin us up and what the total on the dice are?

    (c) How many different PIN numbers are there if each PIN is 4 digits long and digits may be repeated?

    (d) How many different PIN numbers are there if each PIN is 4 digits long and digits may **not** be repeated?