Math 102
Project 1 Handout
Due: 2/08/2008

Instructions: This project is designed to give you an opportunity to explore some of the concepts from set theory. Complete as much of this project as you can by the due date (Friday February 8th). You should write up your solutions neatly and all pertinent work leading up to your solution should be included as well. If you consult any references (books or online material), cite the relevant sources either in footnotes, or at the end of your project.

1. (3 points each) Use Venn diagrams to decide whether or not the following statements are true. If a statement is false, give a specific counterexample that shows that it cannot be true.

   (a) $(A \cup B)' = A' \cup B'$

   (b) $A' \cap B' = (A \cup B)'$

   (c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (d) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2. (5 points) Recall that we proved the equation $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ in class. Come up with a similar equation that allows us to compute $n(A \cup B \cup C)$ in terms of other combinations of A, B, and C. Give a specific example that demonstrates that your formula is reasonable.

3. (3 points) Draw a Venn diagram showing all possible 16 regions for four sets, A, B, C, and D.

4. (5 points) Let $A = \{a, b, c, d, e, f, g, h\}$

   (a) Use Pascal's triangle to find the number of subsets of A with exactly 3 elements.

   (b) Use Pascal's triangle to find the number of subsets of A with exactly 5 elements.

   (c) Explain why the numbers you found in parts (a) and (b) are the same.

   (d) Explain, in your own words, why Pascal's triangle is "symmetric" about its "middle" (vertically).