Math 102
Project 2 Handout
Due: 2/29/2008

Instructions: This project is designed to give you an opportunity to explore some additional concepts from mathematical logic. Complete as much of this project as you can by the due date (Friday February 29th). You should write up your solutions neatly and all pertinent work leading up to your solution should be included as well. If you consult any references (books or on line material), cite the relevant sources either in footnotes, or at the end of your project.

1. (8 points) Given the argument:

   $\sim s$
   $p \to r$
   $(p \to q) \to s$

   $\therefore r$

   Write a two column proof that demonstrates that this argument is valid.

2. (3 points each) The following logic puzzles are based on the work of logician Raymond Smullyan. These problems are all centered around a mythical island which has two types of inhabitants: *knights*, who always tell the truth, and *knaves*, who always lie. Each part below describes an encounter with two inhabitants of the island. Each part should be viewed as independent from the other parts – only use the basic rules and the information given in that part. Your job is to use the information determine the identity of the inhabitants mentioned in each part. That is, to determine for each inhabitant mentioned whether he is a knight or a knave. You must write out a proof in paragraph form which proves the identity the inhabitants in each problem.

   (a) $\mathbb{A}$ says "At least one of us is a knave". $\mathbb{B}$ says nothing.

   (b) $\mathbb{A}$ says "Both of us are knights". $\mathbb{B}$ says "$\mathbb{A}$ is a knave".

   (c) $\mathbb{A}$ says "I am a knave or $\mathbb{B}$ is a knight". $\mathbb{B}$ says nothing.

   (d) $\mathbb{A}$ says "We are both knaves". $\mathbb{B}$ says nothing.

3. (3 points each) Lewis Carroll, the author of *Alice in Wonderland*, was also a logician. He created many logic puzzles. In this problem, we will be looking at some of his "syllogism puzzles". In these puzzles your job is to write out each of the given premises symbolically as conditional statements. Then write out the contrapositives of each statement. Next, string the symbolic conditional statements together to form the longest chain of syllogisms possible. Finally, you should translate the first hypothesis and final conclusion as a single conditional statement written in ordinary English.

   (a) All unripe fruit is unwholesome. All these apples are wholesome. No fruit, grown in the shade, is ripe.

   (b) Colored flowers are always scented. I dislike flowers that are not grown in the open air. No flowers grown in the open air are colorless.

   (c) All writers, who understand human nature, are clever. No one is a true poet unless he can stir the hearts of men. Shakespeare wrote "Hamlet". No writer, who does not understand human nature, can stir the hearts of men. None but a true poet could have written "Hamlet".