Math 102
Project 3 Handout
Due: By 4:00pm on Monday 4/07/2008

Instructions: This project is designed to give you an opportunity to explore some additional concepts from counting and probability. Complete as much of this project as you can by the due date (Monday April 7th). You should write up your solutions neatly and all pertinent work leading up to your solution should be included as well. Make sure that you show enough detail in your computations that I can see how you arrived at your answers. If you consult any references (books or online material), cite the relevant sources either in footnotes, or at the end of your project.

1. Remember the slot machine from a previous worksheet whose first wheel has 3 cherries, 5 oranges, 2 bars, 4 bells, and 6 pears, second wheel has 5 cherries, 7 oranges, 4 bars, 1 bell, and 3 pears, and third wheel has 1 cherry, 6 oranges, 2 bars, 3 bells, and 8 pears.

   (a) (4 points) Recall that to win on this slot machine, you need to get 3 matching symbols when the wheels come to rest. Compute the probability of getting 3 Cherries, the probability of getting 3 Oranges, the probability of getting 3 Bars, the probability of getting 3 Bells, and the probability of getting 3 Pears.

   (b) (4 points) It costs $1 for each spin on this slot machine. The payouts for this slot machine are as follows: 3 Oranges pays $5, 3 Pears pays $10, 3 Cherries pays $100, 3 Bars pays $100, and 3 Bells pays $100. Find the expected value for playing this slot machine [Hint: What is the probability of losing?].

   (c) (4 points) Suppose you had the ability to break into the machine when no one was around and rig the first wheel so that one of its 20 spaces is twice as likely to come up as the other 19 spaces. Pick the symbol to come up twice as often that you think would increase the potential winnings for this slot machine the most. Given this "new" setting for the slot machine, find the new probabilities for each of the 3 matching symbol "jackpots" and find the new expected value for the slot machine. Is the expected payout higher or lower than before? Would a casino make money or lose money with this modified machine?

2. The "Powerball" multi-state lottery works as follows: 55 white balls with the numbers 1 through 55 printed on them are placed in a drum and 5 of them are drawn (without replacement). Next, 42 red balls with the numbers 1 through 42 are placed in a different drum and a single ball is drawn (the "power ball"). To play, before the drawing, and player pays $1 for the right to try to predict the results of the drawing by choosing 5 "white" numbers plus a "power ball" number.

   (a) (2 points) Find the total number of possible outcomes for the Powerball drawing process.

   (b) (2 points) A player wins the jackpot by correctly picking all 5 white numbers **and** the red Powerball number. Find the probability of winning the jackpot.

   (c) (8 points) Find the probability of the following other ways to win at Powerball (you must show your work in order to receive credit for this part of the project):
      i. Correctly picking all 5 white numbers (successfully doing this pays $200,000)
      ii. Correctly picking exactly 4 white numbers **and** the Powerball (successfully doing this pays $10,000)
      iii. Correctly picking exactly 4 of the white numbers (successfully doing this pays $100)
      iv. Correctly picking exactly 3 white numbers **and** the Powerball (successfully doing this pays $100)
      v. Correctly picking exactly 3 of the white numbers (successfully doing this pays $7)
      vi. Correctly picking exactly 2 white numbers **and** the Powerball (successfully doing this pays $7)
      vii. Correctly picking exactly 1 white number **and** the Powerball (successfully doing this pays $4)
      viii. Correctly picking the Powerball (successfully doing this pays $3)

   (d) (3 points) The current jackpot value is $36 million. Find the expected value for playing Powerball given the current jackpot and the payouts listed above.

   (e) (3 points) How big would the jackpot have to be in order to make playing Powerball a fair game?