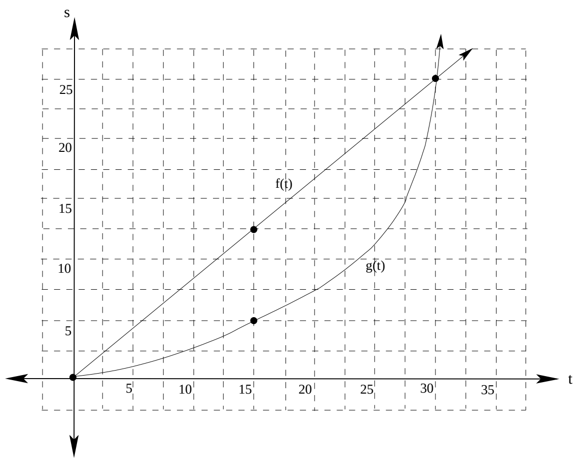
Math 261

Exam 2 - Practice Problems

1. Find the derivative $y' = \frac{dy}{dx}$ for each of the following:
   (a) $y = e^2x + ex^2$
   (b) $y = \cot x$
   (c) $y = \sqrt{x}\sec(x^2)$
   (d) $y = 2\tan^3(2x^3)$
   (e) $y = \dfrac{x^2 - 7\cos(3x)}{x + \sin(3 - 2x)}$
   (f) $x^2y + 3xy - 5y^2 = 7$
   (g) $\cos^2(xy) = 1$

2. Use the formal limit definition of the derivative to find the derivative of the following:
   (a) $f(x) = x^2 - 3x$
   (b) $f(x) = \dfrac{2}{x-3}$
   (c) $f(x) = \sqrt{x-2}$

3. The position of two cars, car A and car B, both starting side by side on a straight road, is given by $f(t)$ and $g(t)$, where $f(t)$ is the distance traveled car A in feet, and $g(t)$ is the distance traveled car B in feet, and t is in minutes (see the graph below):



   (a) How fast is car A going at time $t = 15$?
   (b) Find the average rate of change of car B on the time interval $[0, 15]$.
   (c) Which car is traveling faster at time $t = 15$?
   (d) Which car is traveling faster at time $t = 30$?
   (e) What can you say about the relative positions of the two cars at time $t = 30$?

4. Use the quotient rule to derive the formula for the derivative of $\tan(x)$.
5. Use the product rule to prove that $D_x[f(x)g(x)h(x)] = f'(x)g(x)h(x)+f(x)g'(x)h(x)+f(x)g(x)h'(x)$
6. If $f(x) = \sqrt{3x-5}$, find the intervals where $f(x)$ is continuous, and find the intervals where $f(x)$ is differentiable.
7. If $f(x) = 3x^4 - 5x^2 + 7x - 12$, use differentials to approximate $f(1.1)$
8. Use differentials to approximate $\sqrt{1.2}$. How good is your approximation?
9. Use differentials to estimate $\sqrt[3]{9}$. How good is your approximation?
10. Suppose helium is being pumped into a spherical balloon at a rate of 4 cubic feet per minute. Find the rate at which the radius is changing when the radius is 2 feet.
11. Dr. Von Klausen has just invented a shrink ray and decides to try it out on a test object: a cylinder whose height is twice its radius. The shrink ray has been calibrated so that the proportions of the cylinder remain the same throughout the test. A few seconds into the test, the radius of the cylinder is decreasing at 2 inches per second, and the height is 4 inches. At what rate is the volume of the cylinder changing at that time? (Be sure to include units in your answer)
12. A company manufactures wooden cubes. Each side of the finished cubes are 5 inches long, with a maximum error of $\pm.2$ inches per side. Use differentials to estimate the maximum error in the volume of the cube. Then, compare your estimate with the error in volume of a cube with largest possible volume manufactured within the given error tolerances.
13. Find the equation of the tangent line to the graph of $f(x) = \tan(4x)$ when $x = \frac{3\pi}{16}$
14. Find the equation of the tangent line to the graph of $y = \sec(2x)$ when $x = \frac{\pi}{6}$.
15. Find the points on the graph of $y = 2x^3 + 3x^2 - 72x + 5$ at which the tangent line is horizontal.
16. Find the equation of the tangent line to the graph of the relation $x^2y + 3y^2 = 3x - 7$ at the point $(2, -1)$
17. Draw the graph of a function $f(x)$ that is continuous when $x = 3$, but is not differentiable when $x = 3$.
18. Find $g'(2)$ if $h(x) = f(g(x))$, $f(3) = -2$, $g(2) = 3$, $f'(3) = 5$, and $h'(2) = -30$.
19. Given that $f(2) = -3$, $g(2) = 2$, $f'(2) = \frac{1}{2}$, $g'(2) = -5$, and $h(x) = f(g(x))$.

    Find the following:

    (a) $(f-g)'(2)$

    (b) $(fg)'(2)$

    (c) $\left(\frac{f}{g}\right)'(2)$

    (b) $h'(2)$
20. Find $f^{(8)}(x)$ if $f(x) = \sin(2x)$
21. Find $f^{(13)}(x)$ if $f(x) = x^{12} + 7x^5 - 3x^3 - 1$