

Math 261  
Final Exam Review Sheet

**Final Exam: 3:00pm on Thursday, May 1st in Bridges Room 261**

**Review Material**

- Piecewise defined functions, even and odd functions, key values of trig functions

**Limits and Continuity**

- Be able to investigate limits by constructing a table of values.
- Be able to evaluate limits using algebra, using graphs, and using the properties of limits.
- Know how to find one - sided limits and limits of piece-wise defined functions.
- *Memorize* the formal  $\epsilon$ - $\delta$  definition of a limit.
- Given a function and a specific  $\epsilon$  value, be able to find the related  $\delta$  value.
- Know how to prove a limit using the formal definition or prove a limit does not exist using the definition.
- Understand limits as  $x \rightarrow \pm\infty$  and one and two sided limits where the function value approaches to  $\pm\infty$
- Know the definition of continuity of a function at a point  $x = c$
- Be able to classify points of discontinuity as removable, jump, or infinite discontinuities.
- Understand continuity on open and closed intervals:  $(a, b)$  and  $[a, b]$
- Know results about the limits of polynomial, rational functions, roots of functions and combinations of functions.
- Be able to find the points of discontinuity of a given function.
- Know the statement and basic applications of the Intermediate Value Theorem.

**Tangent Lines, Rates of Change, and the Derivative**

- Understand secant lines, finding the average rate of change over an interval, tangent lines and finding the instantaneous rate of change at a point.
- Memorize the formal limit definition of the derivative of a function and be able to use it to find the derivative of a function.
- Know how to use the derivative to find the equation of a tangent line to a function.
- Understand the definition of differentiability on open and closed intervals and be able to find where a function is differentiable.
- Understand right and left hand derivatives, vertical tangent lines and cusps.
- Remember that differentiable functions are continuous, but continuous functions may not be differentiable.
- Memorize the differentiation formulas for constant functions, power functions, the 6 trig functions, differentiation rules for sums, differences, products, and quotients of functions, and the chain rule and be able to apply these rules to find the derivatives of various functions.
- Be able to solve application problems involving tangent lines and rates of change and proofs of trigonometric differentiation formulas using the quotient rule.

**Applications of the Derivative**

- Know how to find the derivative of an implicit function and how to find equations of tangent lines to points on implicit curves.
- Know how to use implicit differentiation to solve *related rates problems*.
- Know the definitions of increasing, decreasing, constant, maximum value, minimum value, local maximum, and local minimum and be able to apply them to the graph of a function.
- Understand the connection between critical numbers and local extrema and be able to find the critical numbers of a given function.
- Know the statement of the Extreme Value Theorem and be able to apply it to find the extrema of a continuous function on a closed interval.
- Memorize the statements of the Mean Value Theorem (MVT) and the Intermediate Value Theorem (IVT) and be able to determine whether a given function satisfies the hypotheses of the MVT and/or the IVT.
- Be able to apply the MVT and/or IVT to reach conclusions about a function given via a formula or a table of values.
- Understand the connection between the sign of the derivative and the increasing/decreasing behavior of a function.
- Know how to find the intervals where a function is increasing/decreasing and classify its local extrema by analyzing the first derivative of the function or by using the second derivative test.
- Understand the connection between the concavity of a function and the sign of the second derivative of the function and know how to find the intervals where a function is concave up/down and find any inflection points by analyzing the second derivative of the function.
- Be able to reach conclusions about the shape of the graph of a function by looking at the graph of its first or second derivative.

- Understand how to find the vertical and horizontal asymptotes of a function as well as the intercepts of a function.
- Be able to analyze the first and second derivatives of a function in order to find the intervals where it is increasing/decreasing, concave up/down, and to find the coordinates of all local extrema and inflection points.
- Be able to use limits to find the vertical and horizontal asymptotes of a function
- Be able to combine information about asymptotes, intercepts, local extrema, inflection points, increasing/decreasing intervals, and concavity in order to draw an accurate graph of a function.
- Understand the idea of *optimization* and how to use information from the first and second derivative in order to find maximum and minimum values for a given function under certain constraints and be able to apply these methods to specific situations.
- Understand the relationship between position, velocity, and acceleration and be able to find equations for position, velocity, and acceleration satisfying given initial conditions.
- Understand how to use Newton's method to approximate a zero of a function and be able to use Newton's method to approximate  $n$ th roots of numbers.

### Antiderivatives and Indefinite Integrals and Definite Integrals

- Know the definition of an antiderivative of a function and how to find the indefinite integral of a function, including the arbitrary constant of integration.
- Know how to solve differential equations by using initial conditions to solve for constants of integration.
- Know how to carry out a change of variables for definite and indefinite integrals.
- Understand summation notation and the basic properties of sums.

• Memorize the summation formulas:  $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ ,  $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ , and  $\sum_{k=1}^n k^3 = \left[ \frac{n(n+1)}{2} \right]^2$  and be able

to apply them to find the values of given sums.

- Understand how to use rectangular polygons to approximate the area under a continuous function on an interval  $[a, b]$  using left hand, right hand, and midpoint heights.
- Understand how to use sums and limits to find the exact area under a continuous function on an interval  $[a, b]$ .
- Understand the definition of a partition  $P$  of a closed interval  $[a, b]$  and the definition of the norm of a partition.
- Understand how to find the Riemann sum area approximation associated with a particular partition.
- Know the definition of the definite integral of a function  $f$  on an interval  $[a, b]$  (i.e. the limit of Riemann sums on the interval as the norm  $\|P\| \rightarrow 0$ , provided the limit exists).
- Understand the connection between the definite integral and area under a function, the fact that continuous functions are integrable, and that not every function is integrable on a given interval.
- Know the properties of definite and indefinite integrals and be able to apply them.
- Memorize the Mean Value Theorem for definite integrals and be able to apply it to a specific function on a given interval.
- Be able to find the average value of a function on a given interval.
- Memorize both parts of the statement of the fundamental theorem of calculus.
- Be able to apply the fundamental theorem of calculus to evaluate definite integrals of continuous functions and to do computations involving both integration and differentiation.
- Understand the properties of definite integrals of even and odd functions on intervals of the form  $[-a, a]$ .
- Understand the relationship between differentiation and definite and indefinite integrals.
- Know how to estimate a definite integral using the Trapezoid Rule and using the Simpson's Rule.
- Know how to estimate the error for approximations of definite integrals when using the Trapezoid Rule and Simpson's Rule.

### Conic Sections

- Memorize the basic formulas for parabolas, ellipses and hyperbolas.
- Understand the geometric properties of the graphs of parabolas, ellipses and hyperbolas.
- Be able to graph a conic section from its equation and be able find its equation given some of its geometric properties.
- Be able to solve application problems involving conic sections.