Math 261
Integration Handout 2
04/07/2007

Properties of the Definite Integral

Definite Integrals

Approximating Area Using Partitions:

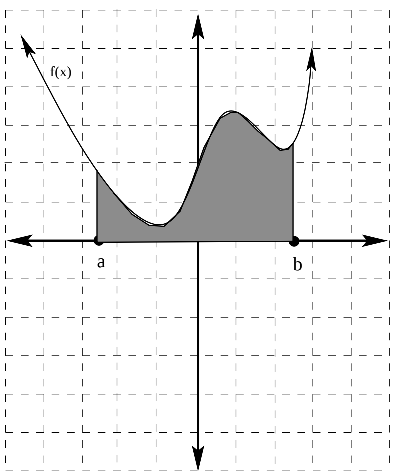
Given a function f on an interval $[a, b]$, we can approximate area using partitions that do not necessarily have rectangles all of the same width. A *partition* P of the interval $[a, b]$ of size n is a set of numbers $a = x_0 < x_1 < x_2 < ... < xn - 1 < x_n = b$. $\Delta x_k = x_k = x_{k-1}$ is the width of the kth subinterval, and $\|P\|$, the *norm* of the partition P, is the width of the widest of all the subintervals in P.
The *Riemann sum* of f on $[a, b]$ for a partition P is $R_P = \sum_{k=1}^{n} f(w_k)\Delta x$, where w_k is some point in the kth subinterval of the partition P.
If $\lim_{\|P\|\to 0} \sum_{k=1}^{n} f(w_k)\Delta x = L$ for some real number L, then we say that f is integrable on $[a, b]$, and the definite integral of f on $[a, b]$ is:

$$\int_a^b f(x)dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(w_k)\Delta x = L$$

The Fundamental Theorem of Calculus:

Let f be a continuous function on the interval $[a, b]$:
(a) If G is the function defined by $\int_a^x f(t)\ dt$ for every x in $[a, b]$, then G is an antiderivative of f on $[a, b]$.
(b) If F is any antiderivative of f on $[a, b]$, then:

$$\int_a^b f(x)\ dx = F(b) - F(a)$$



Example: $\int_1^3 3x^2\ dx = x^3 \Big|_1^3 = 3^3 - 1^3 = 27 - 1 = 26$

Properties of Definite Integrals

1. $\int_a^b c\ dx = c(b-a)$

2. $\int_a^a f(x)\ dx = 0$

3. $\int_a^b f(x)\ dx = -\int_b^a f(x)\ dx$

4. $\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$, for any constant c

5. $\int_a^b f(x) \pm g(x)\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$

6. $\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$

7. If f is integrable on $[a,b]$ and $f(x) \geq 0$ for every x in $[a,b]$, then $\int_a^b f(x)\ dx \geq 0$

8. If f and g are integrable on $[a,b]$ and $f(x) \geq g(x)$ for every x in $[a,b]$, then $\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$

The Mean Value Theorem for Definite Integrals:

If f is continuous on $[a,b]$, then there is a number z in the open interval (a,b) such that $\int_a^b f(x)\ dx = f(z)(b-a)$

The Average Value of a Function

Let f be a function that is integrable on an interval $[a,b]$. Then the **average value** of f over $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\ dx$.