

Recall: The basic steps to solve related rates problems are:

- a) Read the problem and organize the key facts and quantities described.
- b) Draw and label a diagram representing the situation and introduce variables for unknown quantities.
- c) Express all known facts and rates in terms of the variables introduced and their derivatives.
- d) Formulate a general equation relating the variables.
- e) Differentiate the general equation implicitly, yielding the general relationship between the rates.
- f) Substitute any known values and rates, then use algebra to solve to find the unknown rate of change.

Solve the following related rates exercises:

1. The width of a rectangle is half its length. At what rate is its area increasing if its width is 10cm and is increasing at $0.5 \frac{cm}{sec}$?

2. Sand being emptied from a hopper at the rate of $10 \frac{ft^3}{sec}$ forms a conical pile whose height is always twice its radius. At what rate is the radius of the pile increasing when its height is 5 feet?

3. Water is being collected from a block of ice with a square base. The water is produced because the ice is melting in such a way that each edge of the base of the block is decreasing at 2 inches per hour, while the height of the block is decreasing at 3 inches per hour. What is the rate of the flow of water into the collecting basin when the base has an edge length of 3 feet, and the block is 3 feet tall? (You may assume that water and ice have the same density).
4. A balloon is 200 feet off the ground and rising vertically at the constant rate of 15 feet per second. An automobile passes beneath the balloon travelling along a straight road at the constant rate of 45 mph. How fast is the distance between them changing 1 second later?

5. Two ships are sailing toward the same small island. One ship, the *Pinta*, is east of the island and is sailing due west at 15 miles per hour. The other, the *Niña*, is north of the island and is sailing due south at 20 miles per hour. At a certain time, the *Pinta* is 30 miles from the island, and the *Niña* is 40 miles from the island. Are the ships drawing closer together or farther apart? At what rate?
6. Suppose a triangle has sides a and b , and angle θ between the sides a and b . Suppose a is currently 10 feet, b is 15 feet, and θ is $\frac{\pi}{3}$ radians. Also, at this instant a is increasing at 3 feet per second, b is decreasing at a rate of 2 feet per second, and θ is decreasing at a rate of $\frac{2}{25}$ radians per second. What is the rate of change of the area of the triangle at this moment?