Note: While this lab is organized roughly in order of increasing difficulty, to make efficient use of class time you probably want to start with #5.

1. Find all numbers c that satisfy the conclusions of the Mean Value Theorem for the following functions on the given intervals. In part (1b), approximate your answer to the nearest thousandth of a radian.

   (a) $f(x) = 3x^2 - 2x + 15$ on $[0, 5]$

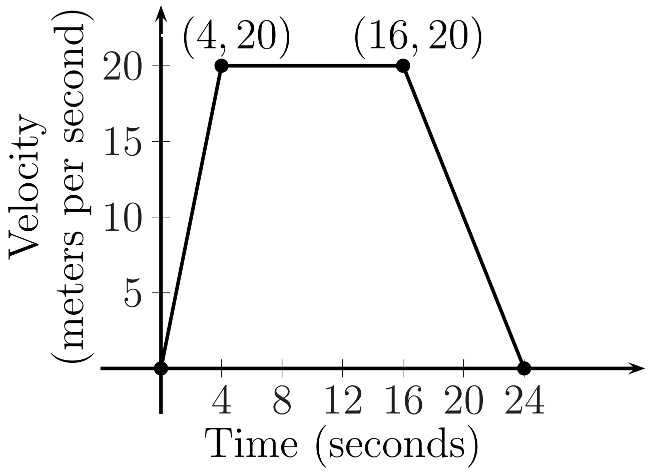
   (b) $f(x) = \sin(x)$ on $\left[0, \dfrac{3\pi}{2}\right]$

2. (From the 2007 AP Calculus AB exam.) The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32 °F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$. For the following, give your answers rounded to three significant digits and use appropriate units.

   (a) Find the average rate of change of W over the interval $5 \le v \le 60$.

   (b) Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

3. (From the 2005 AP Calculus AB exam.) A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph below.



(a) Either find each of the following values or explain why it does not exist. Indicate the units of measure.

(i) $v'(4)$

(ii) $v'(20)$

(b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. Note that in terms of the functions, the acceleration is the derivative of the velocity. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(c) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Indicate teh units of measure. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

4. (From the 2006 AP Calculus AB exam) A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. Recall that the acceleration is the derivative of the velocity with respect to time. The table below shows selected values of these functions.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

(a) For $0 < t < 60$, must there be a time when $v(t) = -5$? Justify your answer.

(b) For $0 < t < 60$, must there be a time when $a(t) = 0$? Justify your answer.

5. (From the 2007 AP Calculus AB exam.) Assume that the functions f and g are differentiable for all real numbers, and that g is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value k for $1 < k < 3$ such that $h'(k) = -5$.

(c) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(d) Explain why the condition that g is increasing is necessary in part (5c).