1. The height of a projectile fired from ground level at an angle θ from the horizontal and at a velocity of v_0 is often approximated by

$$y(t) = v_0 t \sin(\theta) - \frac{1}{2} g t^2$$

where g is the acceleration due to gravity. Its horizontal distance from the point at which it is fired is approximated by

$$x(t) = v_0 t \cos(\theta).$$

Note that g, v_0, and θ are all constants. Use these formulas to answer the following questions.

(a) Find the projectile's vertical velocity as a function of time.

(b) Find the projectile's vertical acceleration as a function of time.

(c) Find the projectile's horizontal velocity as a function of time.

(d) Find the projectile's horizontal acceleration as a function of time.

(e) Find the projectile's overall speed as a function of time.

(f) Find the projectile's overall acceleration as a function of time.

(g) When does the projectile attain its maximum height?

(h) Find the projectile's maximum height.

(i) When does the projectile hit the ground?

(j) Find the range of the projectile (the horizontal distance from the place where it is fired to where it hits the ground).

(k) Let $\theta = 35^\circ$ and $v_0 = 900$ meters per second. Using your formulas from parts (1h) and (1j), find the maximum height and range of the projectile for each of the following bodies from our solar system. Round all of your answers to three significant digits.

i. Earth with $g = 9.81$ meters per second per second.

Height=________________________________

Range=________________________________

ii. Earth's moon with $g = 1.62$ meters per second per second.

Height=________________________________

Range=________________________________

iii. Neptune with $g = 11.4$ meters per second per second. (Yes, Neptune is a gas giant, but *pretend* that it has a surface for the projectile to land on for the purposes of this problem.)

Height=________________________________

Range=________________________________

iv. Saturn's moon Mimas with $g = .0651$ meters per second per second. (Mimas has an absolutely gigantic crater which covers about a quarter of its surface.)

Height=________________________________

Range=________________________________

v. The radius of Mimas is approximately 196,000 meters. Compare this to your previous answer and give a reasonable explanation.

2. Approximate a real root of the following function using Newton's method. Give your answer to six decimal places. Show your work, including your initial guess and all intermediate results.

   $f(x) = 8x^3 - 9x^2 + 12x - 15$

3. Consider $\sqrt{21}$.

   (a) What function would you choose if you wanted to use Newton's method to approximate $\sqrt{21}$?

   (b) Approximate $\sqrt{21}$ using this function. Show your work, including your initial guess and all intermediate results.

4. What single function would you use to approximate $\sqrt{21} + \sqrt{37}$ using Newton's method?