Spring 2008 Math 261 Lab 19

Definite Integrals Name:

1. Assume that 
$$\int_{2}^{5} f(x)dx = 6$$
,  $\int_{-1}^{2} f(x)dx = 9$ ,  $\int_{-1}^{5} g(x)dx = 2$ , and  $\int_{2}^{5} g(x)dx = -8$ . Find  
(a)  $\int_{-1}^{5} f(x)dx$  (c)  $\int_{2}^{5} [3f(x) - 2g(x)] dx$ 

(b) 
$$\int_{-1}^{2} g(x) dx$$
 (d)  $\int_{-1}^{5} [2g(x) + f(x)] dx$ 

2. Find the derivative of each of the following.

(a) 
$$\frac{d}{dx} \int_{3}^{5x^2} \sqrt{4t - 1} dt$$
 (b)  $\frac{d}{dx} \int_{7x - 3}^{3x + 1} \frac{1}{t} dt$ 

3. Evaluate the following definite integrals.

(a) 
$$\int_{-2}^{1} (2x-3) dx$$
 (e)  $\int_{-1}^{0} \frac{8x+22}{(2x^2+11x-5)^2} dx$ 

(b) 
$$\int_{1}^{4} (x\sqrt{x}-2) dx$$
 (f)  $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin(2x)\cos(2x) dx$ 

(c) 
$$\int_{1}^{2} \left(\frac{x-1}{x^{3}}\right) dx$$
 (g)  $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \csc(x) \cot(x) dx$ 

(d) 
$$\int_{3}^{5} \sqrt{2x-5} \, dx$$
 (h)  $\int_{\frac{\pi}{9}}^{\frac{\pi}{9}} \sin\left(x^{2}\right) \, dx$ 

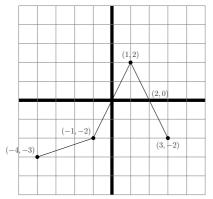
Page 2

Show all Work for Credit

GIVE EXACT ANSWERS UNLESS OTHERWISE NOTED

4. (From the 2005 AP Calculus AB exam.) The graph of the function f below consists of three line segments.

Name:



(a) Let g be the function given by  $g(x) = \int_{-4}^{x} f(t) dt$ . Find the value of each of the following or state that it does not exist.

(i) 
$$g(-1)$$
 (iii)  $g''(-1)$ 

(ii) 
$$g'(-1)$$
 (iv)  $g''(2)$ 

(b) For the function g defined above, find the x-coordinate of each point of inflection of the graph of g on the open interval -4 < x < 3. Show your work and/or reasoning.

5. (Adapted from the 2007 AP Calculus AB exam.) Assume that the functions f and g are differentiable for all real numbers, and that g is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of x. Let w be the function given by  $w(x) = \int_{1}^{g(x)} f(t) dt$ . Find the value of w'(3).

Name:

x	f(x)	f'(x)	g(x)	g'(x)
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

- 6. (From the 2005 AP Calculus AB exam.) The tide removes sand from Sandy Point Beach at a rate modeled by the function R. A pumping station adds sand to the beach at a rate modeled by the function S. Both R(t) and S(t) have units of cubic yards per hour and t is measured in hours for  $0 \le t \le 6$ . At time t = 0, the beach contains 2500 cubic yards of sand.
  - (a) Write an expression for Y(t), the total number of cubic yards of sand on the beach at time t, in terms of R and S.

(b) Let  $R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right)$ . How much sand will the tide remove from the beach during this 6-hour period? Indicate the units of measure and give your answer rounded to the nearest tenth.