

1. (Adapted from the 2005 AP Calculus AB exam.) Consider the curve given by $y^2 = 2 + xy$.

(a) Find $\frac{dy}{dx}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

2. (From the 2007 AP Calculus AB exam.) Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

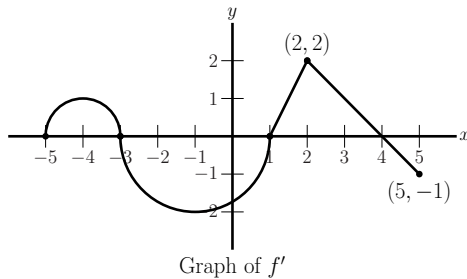
(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

3. (From the 2005 AP Calculus AB exam.) A metal wire of length 8 centimeters (cm) is heated at one end. The table below gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

Distance x (cm)	0	1	5	6	8
Temperature T ($^{\circ}\text{C}$)	100	93	70	62	55

- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- (b) (i) Write an integral expression in terms of $T(x)$ for the average temperature of the wire.
- (ii) Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Round your answer to three significant digits and indicate units of measure. CAUTION: Note that the intervals are not uniform.
- (c) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

4. (From the 2007 AP Calculus AB exam.) Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown below.



- (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
- (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
- (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
- (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.