Math 323 Final Exam Practice Problems

1. Given the vectors $\vec{a} = \langle 1, 2, 3 \rangle$ and $\vec{b} = \langle -1, 1, 2 \rangle$, compute the following:

   (a) $3\vec{a} - 2\vec{b}$

   (b) $\vec{a} \times \vec{b}$.

   (c) A unit vector in the direction opposite $\vec{a}$.

   (d) The component of $\vec{a}$ along $\vec{b}$.

   (e) The projection of $\vec{b}$ along $\vec{a}$.

   (f) The angle between $\vec{a}$ and $\vec{b}$

   (g) A vector which is perpendiclar to $\vec{b}$

2. Evaluate the following limit or show it doesn't exist: $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(2x^2 + 2y^2)}{x^2 + y^2}$

3. A projectile is fired with initial speed $v_0 = 80$ feet per second from a height of 6 feet, and at an angle of $\frac{\pi}{4}$ above the horizontal. Assuming that the only force acting on the obeject is gravity, find its maximum altitude, horizontal range, and speed at impact.

4. Let $f(x, y) = \sqrt{x^2 + y^2}$. Find f_{xx} and f_{yx}.

5. (a) Find the equation of the tangent plane and normal line to the surface $z = \sqrt{x^2 + y^2}$ at the point (3,4,5).

   (b) Use the plane that you found in (a) to estimate the value of z when $x = 4$ and $y = 4$. How good is the approximation?

   (c) Find the direction and magnitude of the maximum rate of change of $z = f(x, y) = \sqrt{x^2 + y^2}$ at (3,4,5).

6. Let $T(x, y) = 3x^2y + xe^y$ denote the temperature of a metal plate at the point (x, y). A thermometer is placed at the point $P = (1, 0)$. At what rate is the temperature changing as the thermometer is moved from P towards the point (2,-3)?

7. Use the Chain Rule to find:

   (a) $g'(t)$ where $g(t) = f(x(t), y(t))$, $f(x, y) = x^2y + y^2$, $x(t) = e^{4t}$, and $y(t) = \sin t$.

   (b) g_u and g_v where $g(u, v) = f(x(u, v), y(u, v))$, $f(x, y) = 4x^2 - y$, $x(u, v) = u^3v + \sin u$, and $y(u, v) = 4v^2$.

8. Use implicit differentiation to find $\frac{dz}{dx}$ if $x^2z - y^2x + 3y - z = -4$.

9. Let $f(x, y) = -\frac{1}{3}x^3 + xy - 12y + \frac{1}{2}y^2$. Find and classify all critical points of $f(x, y)$.

10. Find the maximum value of $f(x, y, z) = x + 2y - 4z$ on the sphere $x^2 + y^2 + z^2 = 21$.

11. Compute a Riemann sum to estimate the volume of the function $f(x, y) = 3x^2 + 4y$ on the region $0 \le x \le 4$, $2 \le y \le 4$ partitioned into $n = 4$ equal sized rectangles, and evaluating each rectangle at its midpoint.

12. Reverse the order of integration in the following iterated integral: $\displaystyle\int_0^9 \int_0^{\sqrt{y}} f(x, y)\, dx\, dy$.

13. Find an iterated triple integral which gives the volume of the solid bounded by the graphs of $x = y^2 + z^2$ and $x = 2z$. DO NOT EVALUATE THE INTEGRAL.

14. Convert the following integral into an iterated integral in spherical coordinates.
DO NOT EVALUATE THE INTEGRAL: $\displaystyle\int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z\, dz\, dy\, dx.$

15. Let $\vec{F}(x, y, z) = \langle 2xy,\, x^2 - 2z,\, 12z - 2y \rangle$.

    (a) Show that $\vec{F}$ is conservative by finding a potential function for $\vec{F}$.

    (b) Evaluate $\displaystyle\int_{(0,0,0)}^{(1,1,2)} \vec{F} \cdot d\vec{r}.$

16. Set up an iterated integral for $\displaystyle\iint_S g(x, y, z)\, dS$ where $g(x, y, z) = x^2 z$ and S is the upper half of the ellipsoid $x^2 + 4y^2 + z^2 = 4$. DO NOT EVALUATE THE INTEGRAL.

17. Use Green's Theorem to evaluate $\displaystyle\oint_C (y^3 + \sin(x^2))\, dx + (x^3 + \cos(y^2))\, dy$, where C is the circle $x^2 + y^2 = 4$ traversed counterclockwise.

18. Let $\vec{F} = \langle y^2 + x,\, y + xz,\, x \rangle$ and S the sphere $x^2 + y^2 + z^2 = 1$. Use the Divergence Theorem to find $\displaystyle\iint_S \vec{F} \cdot \vec{n}\, dS$, where $\vec{n}$ is the outward normal to S at (x, y, z).

19. Use Stokes' Theorem to evaluate $\displaystyle\iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$, where S is the portion of $z = \sqrt{4 - x^2 - y^2}$ above the xy-plane, with $\vec{n}$ upward, and $\vec{F} = \langle zx^2,\, ze^{x+y} - x,\, x\sin(y^2) \rangle$.