Math 102
Conditional Probability

Recall:

1. If an event E is a subset of a sample space S for which all outcomes are equally likely, then $P(E)$, the probability that event E occurs is computed as follows: $P(E) = \dfrac{n(E)}{n(S)}$ where $n(E)$ is the number of outcomes in E and $n(S)$ is the number of outcomes in S.
2. Similarly, the *odds against* the event E are given by $n(E') : n(E)$, where E' is the complement of the event E.
3. If E' is the complement of an event E, then $P(E) + P(E') = 1$. Therefore, $P(E') = 1 - P(E)$.
4. The probability of the union of two events E and F is given by $P(E \cup F) = P(E) + P(F) - P(E \cap F)$. The represents the percent chance of E or F happening for a given experiment.

Conditional Probability:

Definition: Let E and F be events. The probability of event F occurring *given that* event E has *already occurred* is the **conditional probability of F given E**. This is denoted as $P(F|E)$.

Note: To compute $P(F|E)$ we use the following formula: $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$

This makes sense, since if we assume that the event E has already happened, E is our new smaller sample space, and $E \cap F$ is the part of F inside the set E.

Example: Suppose that two fair 6-sided dice (one red and one green) are rolled. Consider the events:
$E = \{$the total showing on the dice is 7 $\}$.

$F = \{$"doubles" are rolled$\}$.

$G = \{$the total on the dice is greater than or equal to 10$\}$.

$H = \{$the green die shows a 5$\}$.

1. Find $P(E)$, $P(F)$, $P(G)$, and $P(H)$.

2. Find $P(F|G)$.

3. Find $P(G|F)$.

4. Find $P(E|H)$.

5. Find $P(H|G)$.

Note: If we rearrange the formula: $P(F|E) = \dfrac{P(E \cap F)}{P(E)}$, we get the formula $P(E \cap F) = P(E) \cdot P(F|E)$.
We can use this formula, along with the idea of a tree diagram, to compute the probability that two or more events occur together.

Examples:

1. Suppose we have a fair coin and we decide to keep flipping it until tails comes up once. What is the probability that we stop after three or fewer flips?

2. Suppose that I have a bag containing 6 red chips, 3 white chips, and 1 blue chip.

   (a) What is the probability of drawing two red chips if I draw them one at a time *with* replacement?

   (b) What is the probability of drawing two red chips if I draw them one at a time *without* replacement?

   (c) What is the probability of drawing two white chips if I draw them one at a time *without* replacement?

   (d) What is the probability of drawing one red chip and one white chip if I draw them one at a time *without* replacement?

Definition: We can also use the idea of conditional probability to determine whether or not two events "depend on each other". We say two events E and F are **independent** if $P(F|E) = P(F)$. Otherwise, we say that E and F are **dependent**.

Examples: Determine whether or not the following pairs of events are independent.

1. $E = \{$a total of 7 is rolled on two fair dice$\}$. $F = \{$a 3 is rolled on the first of the two dice$\}$.

2. $E = \{$a total ≤ 5 is rolled on two fair dice$\}$. $F = \{$a 3 is rolled on the first of the two dice$\}$.