Math 102
Exam 1 **Name:**_____

1. (4 points each) Determine whether each statement below is true or false. If it is true, explain how you know that it is true, and if it is false, give a counterexample that shows it must be false.

   (a) $\dfrac{a}{b+c} = \dfrac{a}{b} + \dfrac{a}{c}$

   False. For example, consider $a = 2$ $b = 1$, $c = 1$. Then $\dfrac{a}{b+c} = \dfrac{2}{1+1} = 1$ while $\dfrac{a}{b} + \dfrac{a}{c} = \dfrac{2}{1} + \dfrac{2}{1} = 4$, and clearly, $1 \neq 4$.

   (b) If $A - B = \emptyset$, then $A \subset B$.

   False. Notice that if $A - B = \emptyset$, then every element of A must also be an element of B. Therefore $A \subseteq B$. Hovever, if $A = B$, then $A - B = \emptyset$, but $A \not\subset B$. Since this was a little tricky, I gave full credit if you accidentally read $\subset$ as $\subseteq$.

   (c) If $A \cap B = \emptyset$ then either $A = \emptyset$ or $B = \emptyset$.

   False. Consider $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. Notice that $A \cap B = \emptyset$, but clearly neither A nor B is empty.

2. (7 points) John has exactly 60 cents worth of change. He has no more than three coins of any one type. All of his coins are either nickels, dimes, or quarters. Find both the LEAST and the GREATEST number of coins that John could have.

   The smallest possible collection is 2 Quarters, and 1 dime: 3 total coins.

   The largest possible collection is 1 Quarter, 2 Dimes, and 3 nickels: 6 total coins.

3. (7 points) In the recent poll, 67.2% of the people surveyed are very concerned about the U.S. economy. Of the people that are very concerned about the U.S. economy, 47% of them believe that the U.S. government can work to end the recession by the end of this year. Given that a total of 1,257 people participated in the poll, give an estimate of the number of people polled who are very concerned about the U.S. economy. Also give an estimate of the number of people who are very concerned about the U.S. economy but who also believe that the government can end the recession by the end of this year.

   First, we estimate the number of people who are very concerned about the U.S. economy. One way to do this is to round the percentage up to 70% and the number of people down to 1200, giving us an estimate of 600 + 120 + 120 = 840 people [70% = 50% + 10% + 10%]

   Next, we estimate the number of people who are very concerned about the U.S. economy but who also believe that the government can end the recession by the end of this year. To do this, first understand that we are taking a percentage of our previous answer, not a percentage of the original number of people surveyed, since we want people who believe *both* statements. One way to do this is to round the percentage up to 50%. Then 50% of 840 people is 420 people.

4. (3 points) Use roster notation to express the set:

   $A = \{\ x \mid x$ is an even number between 5 and 27 that is not the square of a whole number $\}$

   $A = \{6, 8, 10, 12, 14, 18, 20, 22, 24, 26\}$. Notice that we omitted 16 since it is a square number.

5. (3 points) Determine whether or not the following set $A = \{\ x \mid x$ is a whole number that is less than zero $\}$ is well defined. Justify your answer.

   Many of you noticed that since whole numbers are all greater or equal than zero, no elements meet the description. This is not a problem. This set is well defined since we knoe that it is $\emptyset$ (the empty set). Several of you interpreted this as being an infinite set (forgetting that the whole numbers are non-negative). Even under this incorrect assumption, the set is well defined since even though it would have been infinite, we could tell for any specific element whether or not it is in the set.

6. (3 points each)Given that $A = \{\ x \mid x$ is a letter in the word *ridge* $\}$, $B = \{\ x \mid x$ is a letter in the word *girder* $\}$, $C = \{\ x \mid x$ is a letter in the word *bridge* $\}$, $D = \{\emptyset\ \}$, and $E = \emptyset$ indicate whether the following are **True** or **False** [you do NOT need to justify your answers]

   First notice that in roster notation, $A = \{r, i, d, g, e\}$, $B = \{g, i, r, d, e\}$, and $C = \{b, r, i, d, g, e\}$.

   (a) $n(B) = 6$ False. $(n(B) = 5)$ (b) $B \subset C$ True.
   (c) $E \subset D$ True. (d) $A - B = E$ True, since $A = B$.

7. (3 points each) Let $U = \{\ x \mid x$ is a whole number less than 11 $\}$; $A = \{1, 2, 5, 6, 7, 8\}$; $B = \{1, 3, 5, 7, 9\}$; $C = \{2, 4, 5, 6, 7\}$. List each of the following sets in roster notation:

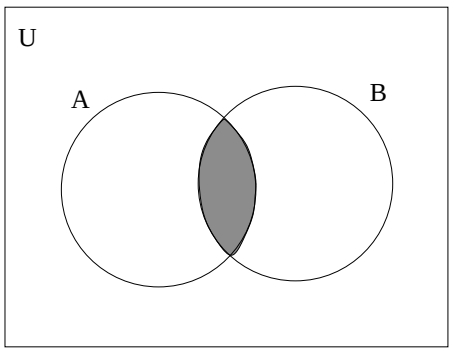
   First notice that $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ [the whole number start from 0].

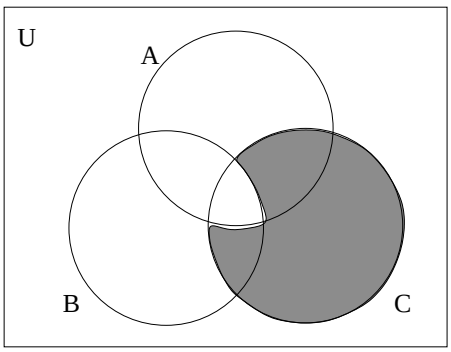
   (a) $B - C = \{1, 3, 9\}$ (c) $(C \cup A') = \{2, 4, 5, 6, 7, \} \cup \{0, 3, 4, 9, 10\} = \{0, 2, 3, 4, 5, 6, 7, 9, 10\}$

   (b) $A' = \{0, 3, 4, 9, 10\}$ (d) $(B \cup C)' = (\{1, 2, 3, 4, 5, 6, 7, 9\})' = \{0, 8, 10\}$

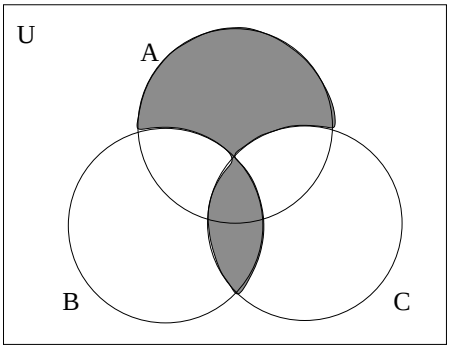
8. (4 points each) Illustrate the following by shading the appropriate regions of the given Venn diagrams:

   (a) $A - B'$ (b) $C - (A \cap B)$





9. (4 points each) Use set notation to describe the shaded regions in each Venn diagram given below:

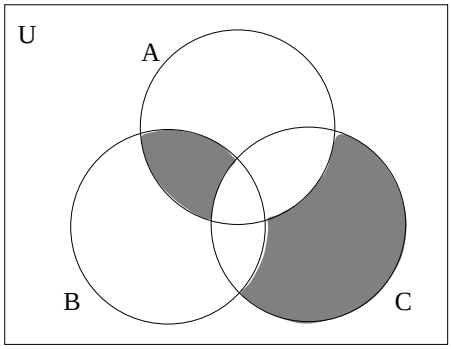


(a)

$(B \cap C) \cup [A - (B \cup C)]$

or

$(A \cup B \cup C) - [(B - C) cup (C - B)]$



(b)

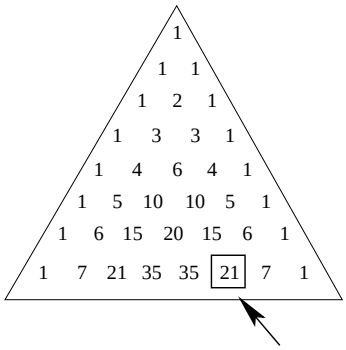
$[(A \cap B) - C] \cup [C - (A \cup B)]$

$[(A \cap B) \cup C] - [(A \cap B) \cup (B \cap C)]$

10. (a) (3 points) How many proper subsets does the set $\{a, b, c, d, e, f\}$ have?

Since this set has 6 elements, it has $2^6 - 1 = 63$ proper subsets.

(b) (5 points) Use Pascal's Triangle to find the number of 5 element subsets of $\{a, b, c, d, e, f, g\}$
Since this set has 7 elements in it, we build Pascal's triangle down to the appropriate row and then find the correct entry for subsets of size 5:



Then there are 21 possible subsets of size 5.

11. (3 points each) Use inductive reasoning to predict the next two terms in the following sequences:

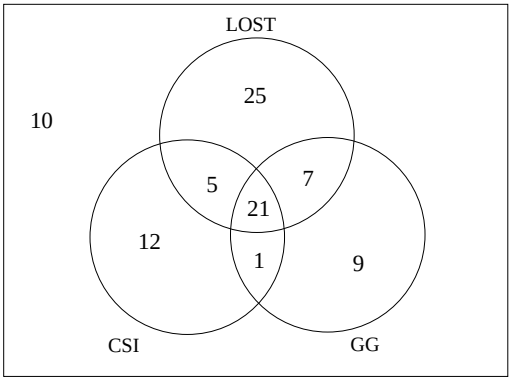
(a) $1, 2, 4, 7, 11, 16, ...$

The pattern here is adding 1, then adding 2, then adding 3, etc.
Therefore, adding $16 + 6 = 22$ and then $22 + 7 = 29$ gives and the next two terms.

(b) $1, 2, 3, 5, 8, 13, 21, ...$

The pattern here is adding the two previous terms to get the next term.
Therefore, adding $13 + 21 = 34$ and then $21 + 34 = 55$ gives and the next two terms.

12. (12 points) A survey asked 90 people about their television viewing habits. Specifically, they were asked which of the following TV programs they watch at least once a month: Lost, CSI, and Gossip Girl. Suppose the survey found that 58 of the people surveyed watch Lost, 9 watch *only* Gossip Girl, 21 watch all three, 28 watch Lost and Gossip Girl, 26 watch Lost and CSI, and 17 watch CSI but *do not* watch Gossip Girl, and 10 watch none of them.



Note that the 1 located between the 12 and the 9 along the bottom of the diagram is the last to be filled in, and is found by taking the total of all of the other regions and subtracting from the 90 people surveyed. The rest could be filled in by carefully applying the information given.

(a) How many people watch CSI?

$5 + 21 + 12 + 1 = 39$ people

(b) How many watch CSI and Lost but *not* Gossip Girl?

5 [read the *and* in the description carefully].

(c) How many do not watch Gossip Girl?

$90 - (21 + 7 + 1 + 9) = 52$

(d) How many watch Gossip Girl and Lost but *not* CSI?

7 [read the *and* in the description carefully].

13. (3 points each)Given p : "puppies are cute", q : "pets are not allowed in my apartment building", r : "I am allergic to pet dander", and s : "I will get a new puppy", translate the following statements into words:

(a) $p \wedge (\sim r)$

Puppies are cute and I am not allergic to pet dander.

(b) $(q \wedge r) \rightarrow (\sim s)$

If pets are not allowed in my building and I am allergic to pet dander, then I will not get a new puppy.