Math 102
Exam 2 - Solutions
02/25/2009

1. (3 points each) Negate each of the following statements. Make sure to write the negation in plain English.

   (a) I always get what I want.

   I sometimes do not get what I want.

   (b) Some people have all the luck.

   No people have all the luck.

   (c) I will go to Mexico for Spring Break and I will get a nice suntan.

   I will not go to Mexico for Spring Break or I will not get a nice suntan.

2. (4 points each) Given p : "I do not like green eggs and ham". q : "I will not try them". r : "I will not eat them here or there". s : "I will not eat them anywhere", translate the following statements into plain English:

   (a) $p \to (r \wedge s)$

   If I do not like green eggs and ham then I will not eat them here or there and I will not eat them anywhere.

   (b) $\sim q \to \sim (r \vee s)$

   If I would try them them then I would eat them here or there and I would eat them anywhere.
   (we used DeMorgan's Law to rephrase the statement. Without this, it would read: If I would try them them then it is not the case that I would not eat them here or there or I not would eat them anywhere).

3. (4 points) Decide whether the "or" used in the following situation is "inclusive" or "exclusive". Make sure to explain your reasoning.

   In order to board the plane, you must show the gate agent a state issued driver's license or a U. S. passport.

   The "or" here is inclusive. To see this, think about what would happen if you showed the gate agent both a state issued driver's license and a passport. Would they still let you board? Clearly yes, so the "or" is inclusive.

4. (3 points each) Given the statements: p : I spent a lot of time working on my project q : I got a good grade on it.

   (a) Write the conditional statement relating p to q in words.

   If I spent a lot of time working on my project then I got a good grade on it.

   (b) Write the contrapositive in words.

   If I did not get a good grade on it then I did not spend a lot of time working on my project.

   (c) Write the converse in words.

   If I got a good grade on it then I spent a lot of time working on my project.

5. (8 points) Use truth tables to determine whether or not the logical statements $p \rightarrow \sim q$ and $\sim (p \wedge q)$ are logically equivalent.

p	q	$\sim q$	$p \rightarrow \sim q)$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

p	q	$p\wedge$	$\sim (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Since the last column in these two truth tables match, the statements are logically equivalent.

6. (7 points) Given that p is true, q is false, r is false, and s is true, what is truth value of the statement: $(p \vee q) \rightarrow (r \wedge \sim s)$ [Note: you do not need to build the entire truth table for this expression]

p	q	r	s	$p \vee q$	$\sim s)$	$r \wedge \sim s$	$(p \vee q) \rightarrow (r \wedge \sim s)$
T	F	F	T	T	F	F	F

Therefore, with the given truth values, this logical expression is False.

7. (7 points each) Define variables and translate each of the following arguments into symbolic form. Then identify the form of each argument and state whether or not the given argument is valid:

(a) If I stay up late on Sunday night, then I will be too tired to go to class on Monday morning. I was too tired to go to class on Monday morning. Therefore, I stayed up too late on Sunday night.

Using the variables: p : I stay up late on Sunday night; q : I am too tired to go to class on Monday morning.

The form of this argument is:

$p \to q$
q
———
$\therefore p$

The is the Fallacy of the Converse, which is **Not Valid**.

(b) If I stay up late on Sunday night, then I will be too tired to go to class on Monday morning. I was not too tired to go to class on Monday morning. Therefore, I did not stay up late on Sunday night.

Using the variables: p : I stay up late on Sunday night; q : I am too tired to go to class on Monday morning.

The form of this argument is:

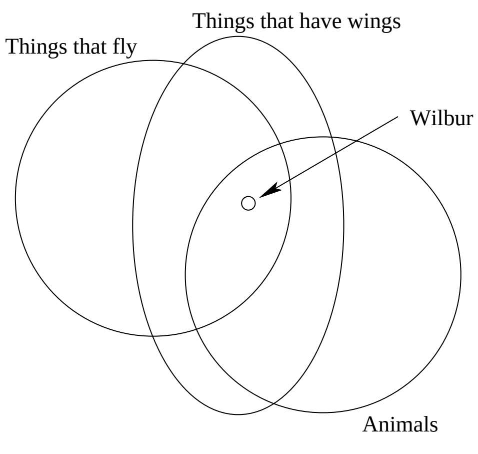
$p \to q$
$\sim q$
———
$\therefore \sim p$

The is the Law of Contraposition, which is **Valid**.

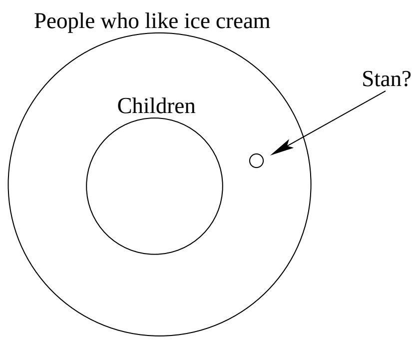
8. (6 points each) Use Euler diagrams to determine whether the following syllogisms are valid or invalid:

(a)
Some animals can fly.
All flying animals have wings
Wilbur can fly
———
Therefore, Wilbur has wings.

(b)
All children like ice cream.
Stan likes ice cream
———
Therefore, Stan is a child.



Valid



Invalid

9. (15 points) Use a truth table to determine whether or not the following argument is valid:

   If I get a raise then I will buy a new car.
   I will buy a new car or I will not take a road trip
   I took a road trip

   Therefore, I got a raise.

   First, we define variables for the concepts involved in this argument:

   Let p be "I get a raise"; q be "I buy a new car", and r be "I take a road trip".

   Then the form of this argument is:

   $p \to q$
   $q \vee \sim r$
   r

   $\therefore p$

   To determine whether or not this argument is valid, we build the truth table for the related expression: $(p \to q) \wedge (q \vee \sim r) \wedge r \to p$

p	q	r	$p \to q$	$\sim r$	$q \vee \sim r$	$(p \to q) \wedge (q \vee \sim r)$	$(p \to q) \wedge (q \vee \sim r) \wedge r$	$(p \to q) \wedge (q \vee \sim r) \wedge r \to p$
T	T	T	T	F	T	T	T	T
T	T	F	T	T	T	T	F	T
T	F	T	F	F	F	F	F	T
T	F	F	F	T	T	F	F	T
F	T	T	T	F	T	T	T	F
F	T	F	T	T	T	T	F	T
F	F	T	T	F	F	F	F	T
F	F	F	T	T	T	T	F	T

   Notice that the last column of the truth table has a False entry. Therefore, this argument is *not valid*.

10. (10 points) Given the argument:

    $q \vee \sim s$
    $\sim r \to p$
    $\sim s \to \sim p$
    $\sim r$

    $\therefore q$

    Fill in the missing statements and reasons in the following two column proof:

1. $\sim r \to p$	Premise
2. $\sim s \to \sim p$	Premise
3. $p \to s$	2, Contraposition
4. $\sim r \to s$	1, 3, Law of Syllogism
5. $\sim r$	Premise
6. s	4, 5, Law of Detachment
7. $q \vee \sim s$	Premise
8. q	6, 7, Disjunctive Syllogism

11. Use basic counting methods to find the following:

(a) (3 points) A restaurant has 3 soups, 7 entrees, and 5 desserts on the menu. How many different meals are possible if a meal consists of one soup, one entree, and one dessert?

Since choosing a dinner corresponds to choosing one of soups, one of 7 entrees and one of 5 desserts, we count the total number of meals by multiplying $(3) \cdot (5) \cdot (7) = 105$ possible meals.

(b) (6 points) Suppose you and I play a game where we each roll a 6 sided die one time. If the number you roll is higher than my roll, then you win. If my roll is equal to or higher than your roll, then I win. How many different pairs of numbers could result from each of us rolling the die once? How many of them would result in you winning?

First, since there are 6 possible results whenever the die is rolled, and we each roll the die once, then we count the total possible number of results by multiplying: $(6) \cdot (6) = 36$ possible pairs of numbers.

Next, to see how often you would win, we look at the outcomes where your roll ends up being higher than mine. There are several ways to do this. One nice way is to build a table to keep track of the results. I will make a table of all 36 possible pairs of numbers that could be rolled and then place an X in each box where you would be the winner:

Your Roll \ My Roll	1	2	3	4	5	6
1						
2	X					
3	X	X				
4	X	X	X			
5	X	X	X	X		
6	X	X	X	X	X	

Therefore, you would win on 15 of the 36 possible pairs of numbers that result from playing this game.