Math 102
Exam 2: Additional Practice Problem Solutions

1. Negate each of the following statements, then rewrite them as English sentences:

   (a) This summer I will get a job or I will take classes.
   It is not the case that this summer I will get a job or I will take classes.
   I will not get a job this summer and I will not take classes this summer.

   (b) If I eat my vegetables then I will get dessert
   It is not the case that if I eat my vegetables then I will get dessert
   I eat my vegetables and I do not get dessert

2. Given p : I studied for this exam, q : I got a good grade on this exam, r : I understand truth tables, and s : I am not good at doing proofs, translate the following statements into words:

3. $p \leftrightarrow (\sim s \vee \sim r)$
   I studied for this exam if and only if I am good at doing proofs or I do not understand truth tables.

4. $(p \rightarrow (r \wedge (\sim s)) \rightarrow q$
   If whenever I study for this exam it is also true that I understand truth tables and I am good at doing proofs then I will get a good grade on this exam.

5. (a) Explain, in your own words, the difference between "exclusive or" and "inclusive or"
   Exclusive or is used to indicate that one of two things is true, but not both.
   Inclusive or indicates that one of two things are true, or both could be true as well.

   (b) Give real world examples that illustrate both "exclusive or" and "inclusive or"
   Exclusive Or: I will drive to work or I will take the bus.
   Inclusive Or: To get all of my work done, I need to stay at work late or go into work early.

6. Given the statements: p : There is a full moon tonight, and q : I will go for a walk on the beach

   (a) Write the conditional statement relating p to q in words.
   ($p \rightarrow q$:) If there is a full moon tonight, then I will go for a walk on the beach.

   (b) Write the converse in words.
   ($q \rightarrow p$:) If I go for a walk on the beach, then there is a full moon tonight.

   (c) Write the inverse in words.
   ($\sim p \rightarrow \sim q$:) If there is a not a full moon tonight, then I will not go for a walk on the beach.

   (d) Write the contrapositive in words.
   ($\sim q \rightarrow \sim p$:) If I do not go for a walk on the beach, then there is not a full moon tonight.

   (e) Indicate which of these statements above are logically equivalent to each other. You do not need to prove your answer.
   The conditional and the contrapositive are logically equivalent.
   The converse and the inverse are logically equivalent.

7. According to one of DeMorgan's Laws, $\sim (p \vee q)$ is logically equivalent to $(\sim p) \wedge (\sim q)$. Use truth tables to prove that these two statements are logically equivalent. Then, explain in your own words why the fact that these two statements are equivalent makes sense.

p	q	$p \vee q$	$\sim (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Since the last column in these two truth tables match, the statements are logically equivalent.

Notice that this makes sense since $\sim (p \vee q)$ means that it is not the case that p or q holds, so we must be in the case where neither one holds, which is what is described by the statement $(\sim p) \wedge (\sim q)$.

8. Given that p is true, q is false, r is true, and s is true:

   (a) What is truth value of the statement: $\sim (p \vee q) \rightarrow (r \wedge \sim s)$
   Solution:

p	q	r	s	$p \vee q$	$\sim (p \vee q)$	$\sim s$	$r \wedge \sim s$	$\sim (p \vee q) \rightarrow (r \wedge \sim s)$
T	F	T	T	T	F	F	F	T

   Therefore, with these truth values, the logical expression is True.

   (b) How many rows would the full truth table for the expression $\sim (p \vee q) \rightarrow (r \wedge \sim s)$ have?
   Solution:
   Since there are 4 variables in the expression, the full truth table would have $2^4 = 16$ rows.

9. Build truth tables for the following logical statements:

   (a) $(p \wedge \sim q) \rightarrow q$

p	q	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \rightarrow q$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

   (b) $\sim q \rightarrow (p \vee \sim r)$

p	q	r	$\sim q$	$\sim r$	$p \vee \sim r$	$\sim q \rightarrow (p \vee \sim r)$
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	T	F	F	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

   (c) $(p \rightarrow q) \leftrightarrow \sim (q \wedge r)$

p	q	r	$p \rightarrow q$	$q \wedge r$	$\sim (q \wedge r)$	$(p \rightarrow q) \leftrightarrow \sim (q \wedge r)$
T	T	T	T	T	F	F
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

10. Identify the form of the following arguments, and state whether the given argument is valid:

(a) If I have enough money saved up, then I will go to Mexico for Spring Break. I did not go to Mexico for Spring Break. Therefore, I did not have enough money saved up.

Solution:

We define p : I have enough money saved up, and q : I will go to Mexico for Spring Break.

Then the argument has the form:

$$\begin{array}{l} p \to q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

This is the Law of Contraposition
Therefore, this argument is valid

(b) If I lie on my tax return, then I will get audited by the IRS. I got audited by the IRS. Therefore, I lied on my tax return.

Solution:

We define p : I lie on my tax return, and q : I get audited by the IRS.

Then the argument has the form:

$$\begin{array}{l} p \to q \\ \underline{q} \\ \therefore p \end{array}$$

This is the Fallacy of the Converse
Therefore, this argument is invalid

(c) I will go to Mexico for Spring Break or I will spend Spring Break with my family. I did not spend Spring Break with my family. Therefore, I went to Mexico for Spring Break.

Solution:

We define p : I will go to Mexico for Spring Break, and q : I will spend Spring Break with my family.

Then the argument has the form:

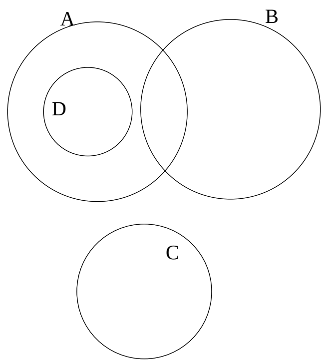
$$\begin{array}{l} p \vee q \\ \underline{\sim q} \\ \therefore p \end{array}$$

This is Disjuctive Syllogism
Therefore, this argument is valid

11. (a) Draw an Euler diagram for the statements: "Some A's are B's", "All C's are not A's", and " All D's are A's"

Solution

There are a few possibilities for this diagram. Here is one of them:



(b) State a valid conclusion that can be made based on the statements in part (a) above.

Solution:

I wanted a bit more than just restating one of the premises here. The main acceptable novel conclusion one can reach based on this Euler diagram is:

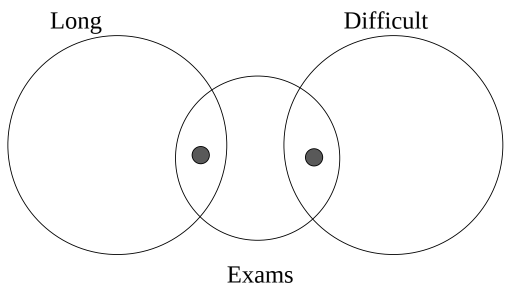
No C's are D's (or All D's are not C's).

12. Use Euler diagrams to determine whether the following syllogisms are valid or invalid:

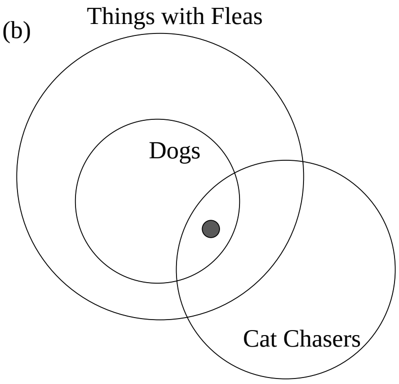
(a) Some exams are too long.
Some exams are too difficult.
Therefore, some exams are too long and too difficult.

(b) Some dogs chase cats.
All dogs have fleas.
Therefore, some cat-chasing dogs have fleas.

(a)



(b)



Invalid

Valid

13. Use a truth table to determine whether or not the following argument is valid:

If I work hard, then I will get a raise.
If I get a raise, then I will not have to get a second job.
I got a second job.
Therefore, I did not work hard.

Solution:

First, we need to translate the argument into logical symbols. To do this, we take p : I work hard, q : I get a raise, and r : I get a second job.

With these variables, the form of this argument is:

$p \to q$
$q \to \sim r$
r
$\therefore \sim p$

With this symbolic representation, to assess the validity of this agrument, we need to investigate the logical expression: $(p \to q) \wedge (q \to \sim r) \wedge r \to (\sim p)$

p	q	r	$\sim r$	$p \to q$	$q \to \sim r$	$(p \to q) \wedge (q \to \sim r) \wedge r$	$\sim p$	$(p \to q) \wedge (q \to \sim r) \wedge r \to (\sim p)$
T	T	T	F	T	F	F	F	T
T	T	F	T	T	T	F	F	T
T	F	T	F	F	T	F	F	T
T	F	F	T	F	T	F	F	T
F	T	T	F	T	F	F	T	T
F	T	F	T	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	T	T	T	F	T	T

Notice that the last column of the truth table is all True entries. Therefore, this argument is valid.

14. Given the argument:

$p \to q$
$\sim (q \wedge r)$
r
$\therefore \sim p$

Fill in the missing reasons in the following two column proof:

Statement	**Reason**
1. $\sim (q \wedge r)$	Premise
2. $\sim q \vee \sim r$	1, DeMorgan's Laws
3. r	Premise
4. $\sim (\sim r)$	3, Double Negation
5. $\sim q$	2, 4, Disjunctive Syllogism
6. $p \to q$	Premise
7. $\sim q \to \sim p$	6, Contraposition
8. $\sim p$	5, 7, Law of Detachment

15. Write a 2-column proof to verify the following argument:

$t \to p$
$s \vee t$
$p \to q$
$\sim q$
$\therefore s$

Solution:

Statement	**Reason**
1. $t \to p$	Premise
2. $p \to q$	Premise
3. $t \to q$	1, 2, Law of Syllogism
4. $\sim q$	Premise
5. $\sim t$	3, 4, Law of Contraposition
6. $s \vee t$	Premise
7. s	5, 6, Disjunctive Syllogism

16. Use basic counting principles to find each of the following:

(a) Suppose I have 5 shirts, 4 pairs of pants, and 3 pairs of shoes. How many different outfits could I wear?
Solution:
We will assume that an "outfit" consists of a shirt, one pair of pants, and one pair of shoes.
Then there are: $5 \cdot 4 \cdot 3 = 60$ different possible outfits.

(b) Suppose I first flip a coin and then roll two 6-sided dice. How many distinct outcomes are there if I only care which side of the coin us up and what the total on the dice are?
Solution:
Notice that there are 11 possible totals when rolling 2 6-sided dice. There there are $2 \cdot 11 = 22$ possible outcomes when first flipping a coin and then rolling two dice and noting the total on the dice.

(c) How many different PIN numbers are there if each PIN is 4 digits long and digits may be repeated?
Solution:
Since there are 10 possible choices of numbers for each digit in the PIN, there are $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$ different PIN numbers.

(d) How many different PIN numbers are there if each PIN is 4 digits long and digits may **not** be repeated?
Solution:
Notice that after each number is chosen to be part of the PIN, that number is "used up" and cannot be used again. Then there are 10 possible choices for the first number, 9 for the second, 8 for the third, and 7 for the fourth number. Therefore, is no repetition is allowed, then there are $10 \cdot 9 \cdot 8 \cdot 7 = 5,040$ possible PIN numbers.