Math 102
Introduction to Probability

Example: Suppose that a fair coin is flipped three times. What is the probability that heads comes up *at least twice*?

Definitions:

1. An **experiment** is any observation of a random phenomenon.
2. The different possible results of an experiment are called **outcomes**
3. The set of all possible outcomes for an experiment is called the **sample space**
4. An **event** is any subset of the sample space.

We will usually use S to denote the sample space and E or F to denote event sets.

In our example stated above, if we use H to represent "Heads" and T to represent "Tails", then the sample space is:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

The event set for heads coming up *at least twice* is:

$S = \{HHH, HHT, HTH, THH\}$

Definitions:

1. The **probability** of an event, denoted $P(E)$ is a number between 0 and 1 (inclusive) that represents the "percentage chance" of that particular outcome occurring. This number can be either *theoretically assigned* or *experimentally assigned*.
2. In the case where all outcomes in the sample space are considered to be equally likely, the probability of an event E is given by the following formula:
   $$P(E) = \frac{n(E)}{n(S)} = \frac{\#\text{ of element in the event set}}{\#\text{ of element in the sample space}}$$
3. The basic properties or probability are:
   (a) $0 \leq P(E) \leq 1$
   (b) $P(\emptyset) = 0$
   (c) $P(S) = 1$

Concluding our example, the probability of getting heads at least twice when flipping a fair coin 3 times is:
$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$ or 0.5.
That is, we expect to get head at least twice half of the time.

Finally, a different way of considering the relative likelihood that a given event occurs is called the "odds" either "in favor of" or "against" the event.

The odds *in favor of* an event E are given by: $n(E) : n(E')$

The odds *against* an event E are given by: $n(E') : n(E)$

Usually, we will reduce this ratio to lowest terms.

In the experiment we performed above, flipping a air coin 3 times, let E be the event of getting heads at least twice, and F the event of getting tails all three times.

Then the odds in favor of E are: $n(E) : n(E') = 4 : 4$, or $1 : 1$

The odds against F are $n(F') : n(F) = 7 : 1$