Math 102 Project 2 - Hint

The following logic puzzle is based on the work of logician Raymond Smullyan. These problems are all centered around an island that has two types of inhabitants: *Verites*, who always tell the truth, and *Perfides*, who always lie. The paragraph below describes an encounter with two inhabitants of this island. Your task is to use the information given to correctly determine the identity of the inhabitants mentioned. That is, to determine for each inhabitant mentioned whether he is a Verite or a Perfide. You must write out a proof in paragraph form which explains how you determined the identity the inhabitants in that part of the problem.

 $\mathbbm{A}$  says: " $\mathbbm{B}$  is a Perfide".  $\mathbbm{B}$  says: "At least one of us is a Verite". Solution:

Suppose that  $\mathbb{A}$  is a Verite. Then his statement " $\mathbb{B}$  is a Perfide" must be true. Therefore,  $\mathbb{B}$ 's statement "At least one of us is a Verite" must be a lie. But that would mean that both of then are Perfides (meaning both of them always lie). This is impossible since we are assuming that  $\mathbb{A}$  is a Verite, so this case is not a logical possibility.

On the other hand, suppose  $\mathbb{A}$  is a Perfide. Then his statement " $\mathbb{B}$  is a Perfide" must be a lie. Then it must be true that  $\mathbb{B}$  is a Verite. As a result,  $\mathbb{B}$ 's statement "At least one of us is a Verite" must be true. In this case, we conclude that  $\mathbb{A}$  is a Perfide and  $\mathbb{B}$  is a Verite.

Therefore, we have shown that the only possibility is that  $\mathbb{A}$  that is a Perfide and  $\mathbb{B}$  is a Verite.

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 $\mathbbm{A}$  says: " $\mathbbm{B}$  is a Perfide".  $\mathbbm{B}$  says: "At least one of us is a Verite".

## Solution:

Suppose that  $\mathbb{A}$  is a Verite. Then his statement " $\mathbb{B}$  is a Perfide" must be true. Therefore,  $\mathbb{B}$ 's statement "At least one of us is a Verite" must be a lie. But that would mean that both of then are Perfides (meaning both of them always lie). This is impossible since we are assuming that  $\mathbb{A}$  is a Verite, so this case is not a logical possibility.

On the other hand, suppose  $\mathbb{A}$  is a Perfide. Then his statement " $\mathbb{B}$  is a Perfide" must be a lie. Then it must be true that  $\mathbb{B}$  is a Verite. As a result,  $\mathbb{B}$ 's statement "At least one of us is a Verite" must be true. In this case, we conclude that  $\mathbb{A}$  is a Perfide and  $\mathbb{B}$  is a Verite.

Therefore, we have shown that the only possibility is that  $\mathbbm{A}$  that is a Perfide and  $\mathbbm{B}$  is a Verite.