

Instructions: This project is designed to give you an opportunity to explore some additional concepts from mathematical logic. Complete as much of this project as you can by the due date (Wednesday March 4th). You should write up your solutions neatly and all pertinent work leading up to your solution should be included. If you consult any references (books or online materials), cite all sources you used either in footnotes, or at the end of your project.

1. (5 points) Given the argument:

$$\begin{array}{l} p \vee \sim r \\ q \rightarrow r \\ \sim p \\ \hline \therefore \sim q \end{array}$$

Write a two column proof that demonstrates that this argument is valid.

2. (3 points each) The following logic puzzles are based on the work of logician Raymond Smullyan. These problems are all centered around an island that has two types of inhabitants: *Verites*, who always tell the truth, and *Perfides*, who always lie. Each part below describes an encounter with two inhabitants of this island. Each part is independent from the other parts – the basic rules remain the same, but the information given applies only to that part. Your task is to use the information given to correctly determine the identity of the inhabitants mentioned in each part. That is, to determine for each inhabitant mentioned whether he is a Verite or a Perfide. You must write out a proof in paragraph form which explains how you determined the identity the inhabitants in that part of the problem.
- (a) A says “B is a Perfide”. B says “Both of us are Verites”.
- (b) A says “At least one of us is a Perfide”. B says nothing.
- (c) A says nothing. B says “We are both Perfides”.
3. (5 points) Lewis Carroll, the author of *Alice in Wonderland*, was also a logician. He created many logic puzzles. In this problem, we will be looking one of his “syllogism puzzles”. In these puzzles your job is to write out each of the given premises symbolically as conditional statements. Then write out the contrapositives of each statement. Next, string the symbolic conditional statements together to form the longest chain of syllogisms possible. Finally, you should translate the first hypothesis and final conclusion as a single conditional statement written in ordinary English.
- (a) Promise breakers are not trustworthy.
Wine drinkers are very communicative.
A man who keeps his promises is honest.
People who do not drink wine are never pawnbrokers.
One can always trust a very communicative person.