Math 262
Final Exam Review Sheet

Part I: Applications of the Integral

• Be able to set and evaluate a definite integral that gives the area of a region enclosed between two functions.
• Be able to find the area of a region bounded by more than two functions or a region bounded by functions that cross each other one or more times.
• Be able to find the volume of a solid formed by revolving a planar region about either a vertical or horizontal line by setting up and evaluating a definite integral consisting of circular cross sections.
• Be able to find the volume of a solid formed by revolving a planar region about either a vertical or horizontal line by setting up and evaluating a definite integral consisting of cross sections in the shape of "washers".
• Be able to find the volume of a solid formed by revolving a planar region about either a vertical or horizontal line by setting up and evaluating a definite integral consisting of cross sections in the shape of "cylindrical shells".
• Be able to find the volume of a solid described by placing cross sections over a planar region each of conform to a given shape (such as a square, rectangle, semicircle, triangle, etc.) by setting up and evaluating a definite integral.
• Be able to find the volume of a solid by interpreting it as a solid over a planar region with cross sections whose area can be computed via a function of one of the coordinate variables by setting up and evaluating a definite integral.
• Be able to find (or estimate) the arc length of a function on a given interval by setting up and evaluating a definite integral.
• You **will not** be tested on finding surface area using definite integrals.
• Be able to find the amount of work needed to perform a given task by setting up and evaluating a definite integral.
• Understand how to find the Moment and center of mass of a system of point masses either along the x-axis or in the xy-plane.
• Understand how to find the mass, Moments and center of mass of a planar lamina of constant density by setting up and evaluating definite integrals.

Part II: Inverse Functions, Exponentials, and Logarithms

• Understand the definition of a one-to-one function and be able to determine whether or not a given function is one-to-one.
• Understand the definition of an inverse function and be able to find an equation for the inverse of a function algebraically.
• Know and be able to apply theorems about continuity and the derivative of the inverse of a function.
• Memorize the definition of the natural logarithmic function and the properties of logarithms.
• Understand the shape and properties of the graph of the natural logarithmic function.
• Be able to compute derivatives of functions involving the natural logarithm and also be able to do logarithmic differentiation.
• Understand the definition of the natural exponential function as the inverse of the natural logarithmic function. Also know the definition of e and the shape of the graph of the exponential function.
• Be able to apply the properties of inverse functions to expressions involving exponential and logarithmic functions.
• Be able to differentiate functions involving exponential functions and evaluate definite and indefinite integrals involving the logarithmic and exponential functions.
• Memorize and be able to apply the integration formulas for all 6 trigonometric functions.
• Be able to rewrite general exponential and logarithmic functions as natural exponential and logarithmic functions.
• Memorize the law of growth/decay (the general solution to the differential equation $\frac{dy}{dx} = ky$).
• Be able to use separation of variables to solve a differential equation and be able to incorporate an initial condition into the general solution to a differential equation.
• Be able solve application problems involving growth, decay, or other related applications.
• Understand the definition of $\arcsin x$, $\arccos x$, $\arctan x$, and $\operatorname{arcsec} x$.
• Understand the domain, range, and graphs of each of the inverse trigonometric functions.
• Be able to compute "key values" of both trigonometric and inverse trigonometric functions *exactly.*
• Memorize the differentiation formulas for the inverse trigonometric functions and be able to apply them in various situations.
• Memorize the integration formulas involving inverse trigonometric functions and be able to apply them in various situations.

Part III: Integration Techniques

• Know and be able to apply integration by parts (along with other techniques) in order to evaluate integrals.
• Understand how to apply integration by parts multiple times in order to evaluate integrals.
• Understand how to rewrite integrands involving trigonometric functions using Pythagorean identities.
• Understand how to rewrite integrands involving trigonometric functions using half angle (power reduction) identities.
• Be able to evaluate various integrals involving powers of $\sin x$ and $\cos x$.
• Be able to use trigonometric substitution in order to solve definite and indefinite integrals.
• Be able to carry out long division of polynomials and know when to apply this procedure to an integrand (when the degree of the numerator of a rational function is greater than or equal to that of the denominator).

• Understand how to find the partial fractions decomposition of a rational function whose denominator factors (for both linear and quadratic factors).
• Understand how to combine long division, partial fractions, algebra, and inverse trigonometric functions to integrate various rational functions.
• Understand how to use completing the square to change the form of an integral involving a quadratic term.
• Be able to recognize which of our previous integration methods can be applied to integrate a rational function after completing the square.
• Be able to use more complicated substitution methods in order to integrate functions.
• *Not tested:* You will not be responsible to use the polynomial substitutions from Theorem 9.6. You also will not be asked to use the "sum to product" identities.
• Understand the hypotheses of L'Hôpital's Rule and be able to evaluate limits using L'Hôpital's Rule.
• Be able to use limits to determine whether or not an improper integral with an infinite limit of integration converges or diverges.
• Be able to use limits to determine whether or not an integral with an infinite integrand [for example, a function on an interval containing a vertical asymptote] converges or diverges.
• Be able to use limits to determine whether or not an integral that has more than one type of improperness converges or diverges.
• Be able to use *comparisons* to determine whether or not an improper integral converges or diverges.

Part IV: Sequences and Series

• Understand what it means for a sequence to be convergent and what it means for a sequence to be divergent.
• Be able to use limits to investigate the convergence/divergence of sequences.
• Know the definition of a monotone sequence and the definition of a bounded sequence. Recall the fact that every bounded monotonic sequence has a limit.
• Be able to use the sandwich theorem to show that a sequence has a limit.
• Understand the definition of an infinite series and the notation used to represent series and their terms.
• Understand and be able to distinguish between the two sequences associated with a given series: the sequences of terms $\{a_n\}$ and the sequence of partial sums $\{S_n\}$.
• Know how to determine whether a telescoping series converges or diverges and when it converges, find its sum.
• Know how to determine whether a geometric series converges or diverges and when it converges, find its sum.
• Understand the proof that the harmonic series diverges.
• Understand and be able to apply the nth term test show that a given series diverges. Also remember that this test **cannot** be used to show that a series converges.
• Understand and be able to apply the theorems about series that are *eventually equal*, and about the effects of addition, subtraction, and scalar multiplication on convergent and divergent series.
• Understand the definitions of positive term series and alternating series.
• Memorize the theorem on p-series and thus be able to determine whether a given p-series converges or diverges.
• Understand and be able to apply the Integral Test, the Basic Comparison Test, the Limit Comparison Test, the Ratio Test, the Root Test, and Alternating Series Test to investigate the convergence/divergence of a given series. Remember to verify any hypotheses needed to use each test. Also, remember the conditions that allow us to detect convergence/divergence and those that lead each test to be inconclusive.
• Understand how to find a bound on the error in approximating the sum of a convergent alternating series by looking at the $n + 1$st term.
• Understand the definition of an absolutely convergent series.
• Given an alternating series, be able to determine whether it converges absolutely, conditionally, or diverges.
• Know the definition of a power series centered at zero, or at some other number c.
• Be able to find the radius of convergence and interval of convergence (including endpoints) for a given power series.
• Memorize the power series representations of the functions: $\frac{1}{1-x}$, $\frac{1}{1+x}$, e^x, $\sin x$, and $\cos x$.
• Be able to find power series representations of other functions by starting with a known power series and then using substitution, differentiation, integration, and/or multiplying by a power function.
• Be able to approximate the value of a function at an x-value within the interval of convergence of a power series representation of the function. Also be able to approximate definite integrals using power series.
• Understand how to find Taylor and Maclaurin Polynomials and Taylor and Maclaurin Series for a given differentiable function.
• Understand how to find the remainder term of a Taylor (Maclaurin) Polynomial.
• Understand the conditions necessary for a function to be represented by its power series and be able to use the remainder term to find an upper bound on the error when approximating a function using its nth Taylor (Maclaurin) polynomial.
• Be able to approximate various quantities using Taylor (Maclaurin) polynomials and be able to find a upper bound on the error in these approximations.