Math 262 Practice Problems Power Series and Taylor Series

1. For each of the following power series, find the interval of convergence and the radius of convergence:

(a) 
$$\sum_{n=1}^{\infty} (-1)^n n^2 x^n$$
  
(b) 
$$\sum_{n=1}^{\infty} \frac{2^n}{n^2} (x-3)^n$$
  
(c) 
$$\sum_{n=1}^{\infty} \frac{n^3}{3^n} (x+1)^n$$
  
(d) 
$$\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{n!} (x-10)^n$$
  
(e) 
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n10^n} (x-2)^n$$

2. Use a known series to find a power series in x that has the given function as its sum:

(a) 
$$x \sin(x^3)$$
  
(b)  $\frac{\ln(1+x)}{x}$   
(c)  $\frac{x - \arctan x}{x^3}$ 

- 3. Use a power series to approximate each of the following to within 3 decimal places:
  - (a)  $\arctan \frac{1}{2}$ (b)  $\ln(1.01)$ (c)  $\sin(\frac{\pi}{10})$
- 4. For each of the following functions, find the Taylor Series about the indicated center and also determine the interval of convergence for the series.
  - (a)  $f(x) = e^{x-1}, c = 1$ (b)  $f(x) = \cos x, c = \frac{\pi}{2}$ (c)  $f(x) = \frac{1}{x}, c = -1$
- 5. For each of the following functions, find the Taylor Polynomial for the function at the indicated center c. Also find the Remainder term.
  - (a)  $f(x) = \sqrt{x}, c = 1, n = 3.$ (b)  $f(x) = \ln x, c = 1, n = 4.$
  - (c)  $f(x) = \sqrt{1+x^2}, c = 0, n = 4.$
- 6. Estimate each of the following using a Taylor Polynomial of degree 4. Also find the error or your approximation. Finally, find the number of terms needed to guarantee an accuracy or at least 5 decimal places.
  - (a)  $e^{0.1}$
  - (b) ln 0.9
  - (c)  $\sqrt{1.2}$