Math 323
Exam 2 - Practice Problems

1. Given the vectors $\vec{a} = \langle 2, 0, -1\rangle$, $\vec{b} = \langle 3, -2, 4\rangle$, and $\vec{c} = \langle 1, -4, 0\rangle$, compute the following:
   (a) A unit vector in the direction of $\vec{c}$.
   (b) The angle between $\vec{a}$ and $\vec{b}$ (to the nearest tenth of a degree).
   (c) $\vec{a} \times \vec{b}$
   (d) The area of the triangle determined by $\vec{a}$, $\vec{b}$, and $\vec{a} + \vec{b}$.
   (e) $comp_{\vec{b}}\vec{a}$.
   (f) A non-zero vector that is perpendicular to $\vec{c}$.
2. Given the vectors $\vec{a} = \langle 1, 2, 0\rangle$, $\vec{b} = \langle -1, 0, 2\rangle$, and $\vec{c} = \langle 0, 1, 1\rangle$, compute the following:
   (a) $2\vec{a} + 3\vec{b}$
   (b) $\vec{a} \cdot \vec{b}$.
   (c) A unit vector in the direction of $\vec{a}$.
   (d) The component of $\vec{c}$ along $\vec{a}$.
   (e) The Volume of the parallelepiped determined by $\vec{a}$,$\vec{b}$ and $\vec{c}$.
   (f) The angle between $\vec{b}$ and $\vec{c}$
3. Decide whether each of the following are true or false. If true, explain why. If false, give a counterexample.
   (a) If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$, then $\vec{b} = \vec{c}$.
   (b) If $\vec{b} = \vec{c}$ then $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$.
   (c) $\vec{a} \cdot \vec{a} = \|\vec{a}\|$
   (d) If $\|\vec{a}\| > \|\vec{b}\|$, then $\vec{a} \cdot \vec{c} > \vec{b} \cdot \vec{c}$
   (e) If $\|\vec{a}\| = \|\vec{b}\|$, then $\vec{a} = \vec{b}$
4. Let P=(2,1,2) and Q=(3,1,1) be points.
   (a) Find parametric equations for a line through these points.
   (b) Find the equation of a plane containing P,Q, and the origin.
5. Given the lines $l_1 : x = 3; y = 6 - 2t; z = 3t + 1$ $l_2 : x = 1 + 2s; y = 3 + s; z = 2 + 2s$:
   (a) Show that the lines intersect by finding the coordinates of a point of intersection.
   (b) Find a vector orthogonal to both lines.
   (c) Find the equation of a plane containing both lines.
   (d) Find the distance of this plane from the origin.
6. Given the two planes $2x + y - z = 4$ and $3x - y + z = 6$
   (a) Find normal vectors to each plane.
   (b) Find parametric equations for the line of intersection of the two planes.
   (c) Find the distance from the origin to the plane $2x + y - z = 4$.
7. Find both the parametric and symmetric equations for the line through the points $P(3, 5, 7)$ and $Q(-6, 2, 1)$.
8. Find an equation of the plane through the point $(1, 4, -5)$ and parallel to the plane defined by $2x - 5y + 7z = 12$.

9. Sketch at least 3 traces, then sketch and identify the surface given by each of the following equations:

   (a) $y^2 + z^2 - x = 0$
   (b) $y^2 + z^2 = 1$
   (c) $4x^2 + 4y^2 - 2z^2 = 4$.
   (d) $4x^2 + y^2 - z^2 = 0$
   (e) $y^2 - 4 = 4x^2 + z^2$
   (f) $x^2 - z = y^2$

10. Let $\vec{r}(t) = \langle t^2, 4 + 3t, 4 - 3t \rangle$ be a vector valued function. Calculate the following:

   (a) $\vec{r'}(t)$
   (b) $\vec{r''}(t)$
   (c) $\int_0^1 \vec{r}(t)dt$
   (d) the arc length of $\vec{r}(t)$ for $0 \leq t \leq 2$. (Just set up the integral, you do not need to evaluate it).
   (e) Find the values of t for which $\vec{r}(t)$ and $\vec{r'}(t)$ are perpendicular.

11. Let $\mathbf{r(t)} = \langle \mathbf{3}, \mathbf{4cos(t)}, \mathbf{4sin(t)} \rangle$.

   (a) Sketch the curve traced out by the vector-valued function $\mathbf{r(t)}$. Indicate the orientation of the curve. What is geometric shape of this curve?
   (b) Find an equation in terms of t for $s(t)$, the arc length of the curve traced out by $\mathbf{r(t)}$ as a function of t.
   (c) Find $\mathbf{r'(t)}$.
   (d) Find an expression for the angle between the vectors $\mathbf{r(t)}$ and $\mathbf{r'(t)}$.
   (e) Find the force acting on a 5 kilogram object moving along the path given by $\mathbf{r(t)}$, in units of meters and seconds (assume that no other forces are acting on this mass).
   (f) Find and draw the position and tangent vectors when $t = \pi$. Find 2 different vectors that are normal to $\mathbf{r}(\pi)$ and $\mathbf{r}'(\pi)$. Draw these vectors (carefully labeled) on the same axes (as the position and tangent vectors).

12. Let $\vec{v}(t) = \langle t^2 - 2t, 4t - 3, t^3 \rangle$.

   (a) Find $\vec{a}(t)$
   (b) If $\vec{r}(0) = \langle 4, -1, 0 \rangle$, find $\vec{r}(t)$.
   (c) Find the force acting on an object of mass 50kg with position function $\vec{r}(t)$ (in units of meters per second).
   (d) Find the speed of the object at time $t = 2$.

13. Suppose that a projectile is launched with initial velocity $v_0 = 100$ ft/s from a height of 0 feet and at an angle of $\theta = \frac{\pi}{6}$.

   (a) Assuming that the only force acting on the object is gravity, find the maximum altitude, horizontal range, and speed at impact of this projectile.
   (b) Find the landing point of this projectile if it weighs 1 pound, is launched due east, and there is a southerly wind force of 4 pounds.

14. Let $f(x, y) = \sqrt{9 - x^2 - y^2}$.

   (a) Sketch the domain of f in the x, y-plane.
   (b) Graph contours for $z = f(x, y)$ for $z = 0, \sqrt{5}$,and $2\sqrt{2}$.

15. Given the function $z = f(x, y) = 1 + x^2 - y$:

   (a) Sketch contours for this function for $z = 0, 1, 2$
   (b) What type of curves are the x-cross sections and the y-cross sections of f?