Math 323
Exam 2 - Practice Problem Solutions

1. Given the vectors $\vec{a} = \langle 2, 0, -1\rangle$, $\vec{b} = \langle 3, -2, 4\rangle$, and $\vec{c} = \langle 1, -4, 0\rangle$, compute the following:

(a) A unit vector in the direction of $\vec{c}$.

$\vec{u} = \frac{\vec{c}}{\|\vec{c}\|} = \frac{\langle 1, -4, 0\rangle}{\sqrt{1+16+0}} = \langle \frac{1}{\sqrt{17}}, \frac{-4}{\sqrt{17}}, 0\rangle$

(b) The angle between $\vec{a}$ and $\vec{b}$ (to the nearest tenth of a degree).

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{(2)(3)+(0)(-2)+(-1)(4)}{\sqrt{4+0+1}\sqrt{9+4+16}} = \frac{2}{\sqrt{5}\sqrt{29}}.$

Then $\theta = \arccos\left(\frac{2}{\sqrt{5}\sqrt{29}}\right) \approx 80.4°$

(c) $\vec{a}\times\vec{b}$

$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -1 \\ 3 & -2 & 4 \end{vmatrix} = \vec{i}(0-2) - \vec{j}(8+3) + \vec{k}(-4+0) = -2\vec{i} - 11\vec{j} - 4\vec{k}$

(d) The area of the triangle determined by $\vec{a}$, $\vec{b}$, and $\vec{a}+\vec{b}$.

$A = \frac{1}{2}\|\vec{a}\times\vec{b}\| = \frac{1}{2}\sqrt{4+121+16} = \frac{\sqrt{141}}{2} \approx 5.937$ square units.

(e) $comp_{\vec{b}}\vec{a}$.

$comp_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|b\|} = \frac{2}{\sqrt{29}}$

(f) A non-zero vector that is perpendicular to $\vec{c}$.

We want $\langle 1, -4, 0\rangle \cdot \langle a, b, c\rangle = 0$, or $a - 4b = 0$.

One way to do this (there are infinitely many choices) is to take $a = 4$. Then $b = 1$ and c can be whatever we choose, so $\vec{v} = \langle 4, 1, 3\rangle$ is a non-zero vector that is perpendicular to $\vec{c}$ (check that the dot product works out).

2. Given the vectors $\vec{a} = \langle 1, 2, 0\rangle$, $\vec{b} = \langle -1, 0, 2\rangle$, and $\vec{c} = \langle 0, 1, 1\rangle$, compute the following:

(a) $2\vec{a} + 3\vec{b} = \langle 2, 4, 0\rangle + \langle -3, 0, 6\rangle = \langle -1, 4, 6\rangle$

(b) $\vec{a}\cdot\vec{b} = (1)(-1) + (2)(0) + (0)(2) = -1$.

(c) A unit vector in the direction of $\vec{a}$.

$\vec{u} = \frac{\vec{a}}{\|\vec{a}\|} = \frac{\langle 1, 2, 0\rangle}{\sqrt{1+4}} = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\rangle$

(d) The component of $\vec{c}$ along $\vec{a}$.

$comp_{\vec{a}}\vec{c} = \frac{\vec{c}\cdot\vec{a}}{\|a\|} = \frac{(1)(0)+(2)(1)+(0)(1)}{\sqrt{5}} = \frac{2}{\sqrt{5}}$

(e) The Volume of the parallelepiped determined by $\vec{a}$,$\vec{b}$ and $\vec{c}$.

$V = \left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = \begin{vmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 1(0-2) - 2(-1-0) + 0(0-0) = -2+2+0 = 0$ (what does this tell you about these vectors?...)

(f) The angle between $\vec{b}$ and $\vec{c}$

$\cos\theta = \frac{\vec{b}\cdot\vec{c}}{\|\vec{b}\|\|\vec{c}\|} = \frac{(0)+(0)+(2)}{\sqrt{1+0+4}\sqrt{0+1+1}} = \frac{2}{\sqrt{5}\sqrt{2}}.$

Then $\theta = \arccos\left(\frac{2}{\sqrt{10}}\right) \approx 50.8°$

3. Decide whether each of the following are true or false. If true, explain why. If false, give a counterexample.

(a) If $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, then $\vec{b}=\vec{c}$.
False. If we take $\vec{a}=\vec{0}$, then $\vec{a}\cdot\vec{b}=\vec{0}$ for **any** vector $\vec{b}$. (in fact, it is still false when we assume all vectors are non-zero. Can you show this?)

(b) If $\vec{b}=\vec{c}$ then $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$.
True. Let $\vec{a}=\langle a_1,a_2,a_3\rangle$, $\vec{b}=\langle b_1,b_2,b_3\rangle$, and $\vec{c}=\langle c_1,c_2,c_3\rangle$ If $\vec{b}=\vec{c}$, then $b_1=c_1$, $b_2=c_2$, and $b_3=c_3$. Then, $\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3=a_1c_1+a_2c_2+a_3c_3=\vec{a}\cdot\vec{c}$.

(c) $\vec{a}\cdot\vec{a}=\|\vec{a}\|$
False. Again let $\vec{a}=\langle a_1,a_2,a_3\rangle$. Then $\vec{a}\cdot\vec{a}=a_1^2+a_2^2+a_3^2=\|\vec{a}\|^2$

(d) If $\|\vec{a}\|>\|\vec{b}\|$, then $\vec{a}\cdot\vec{c}>\vec{b}\cdot\vec{c}$
False. Let $\vec{a}=\langle 0,2\rangle$, $\vec{b}=\langle 1,0\rangle$, and $\vec{c}=\langle 2,0\rangle$. Then $\|\vec{a}\|=2>1=\|\vec{b}\|$, but $\vec{a}\cdot\vec{c}=0$ while $\vec{b}\cdot\vec{c}=2$

(e) If $\|\vec{a}\|=\|\vec{b}\|$, then $\vec{a}=\vec{b}$
False. If $\vec{a}=\langle 1,0\rangle$ and $\vec{b}=\langle 0,1\rangle$, then $\|\vec{a}\|=\|\vec{b}\|=1$, but clearly $\vec{a}\neq\vec{b}$.

4. Let P=(2,1,2) and Q=(3,1,1) be points.

(a) Find parametric equations for a line through these points.
$\overrightarrow{PQ}=\langle 1,0,-1\rangle$, so the line (using P) is given by:
$$\ell:\begin{cases} x=2+t \\ y=1 & t\in\mathbb{R} \\ z=2-t \end{cases}$$

(b) Find the equation of a plane containing P,Q, and the origin.
Let $R=(0,0,0)$. First, we find a normal vector to the plane:
$$\overrightarrow{PQ}\times\overrightarrow{PR}=\langle 1,0,-1\rangle\times\langle -2,-1,-2\rangle=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & -1\\ -2 & -1 & -2\end{vmatrix}=\vec{i}(0-1)-\vec{j}(-2-2)+\vec{k}(-1-0)=-1\vec{i}+4\vec{j}-1\vec{k}$$
Then the plane (using P) has equation: $-1(x-2)+4(y-1)-1(z-2)=0$, or $-x+2+4y-4-z+2=0$, which simplifies to give $x-4y+z=0$

5. Given the lines $l_1: x=3; y=6-2t; z=3t+1$ $l_2: x=1+2s; y=3+s; z=2+2s$:

(a) Show that the lines intersect by finding the coordinates of a point of intersection.
Equating the first pair of coordinate equations for these lines, $3=1+2s$, or $2=2s$, so $s=1$
Then, looking at the third pair of equations with $s=1$, $3t+1=2+2s=4$, so $3t=3$ or $t=1$.
Checking this in the middle pair of equations, $6-2(1)=4$ and $3+1=4$. Hence this pair of lines intersect at the point $(3,4,4)$.

(b) Find a vector orthogonal to both lines.
The vectors associated with this pair of lines are $\vec{v}=\langle 0,-2,3\rangle$ and $\vec{v}=\langle 2,1,2\rangle$ respectively.
To find a vector that is orthogonal (perpendicular) to both, we compute $\vec{v}\times\vec{w}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & -2 & 3\\ 2 & 1 & 2\end{vmatrix}=\vec{i}(-4-3)-\vec{j}(0-6)+\vec{k}(0+4)=-7\vec{i}+6\vec{j}+4\vec{k}$

(c) Find the equation of a plane containing both lines.
We need only find the equation for a plane with normal vector $\vec{n}=-7\vec{i}+6\vec{j}+4\vec{k}$ and containing the point $P(3,4,4)$. This is given by: $-7(x-3)+6(y-4)+4(z-4)=0$, or $-7x+6y+4z-19=0$

(d) Find the distance of this plane from the origin.
Recall that the distance of a point $P(x_0,y_0,z_0)$ from a plane with equation $ax+by+cx+d=0$ is given by:
$$D=\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$
In our case, $P(0,0,0)$, $a=-7$, $b=6$, $c=4$, and $d=-19$, so we have $D=\frac{|-19|}{\sqrt{49+36+16}}=\frac{19}{\sqrt{101}}$.

6. Given the two planes $2x + y - z = 4$ and $3x - y + z = 6$

   (a) Find normal vectors to each plane.
   $\vec{n_1} = \langle 2, 1, -1 \rangle$ and $\vec{n_2} = \langle 3, -1, 1 \rangle$

   (b) Find parametric equations for the line of intersection of the two planes.
   Solving each plane's equation for z, we have $z = 2x + y - 4$ and $z = -3x + y + 6$.
   Then $2x + y - 4 = -3x + y + 6$, so $5x = 10$, or $x = 2$.
   Substituting this into our previous equation, $z = 4 + y - 4$, so $z = y$.
   Thus, if we take $z = t$, we have the line of intersection: $\ell : \begin{cases} x = 2 \\ y = t \\ z = t \end{cases} \quad t \in \mathbb{R}$

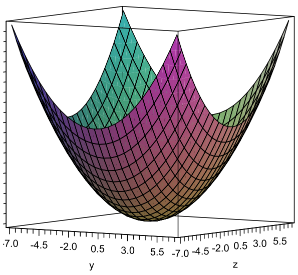
   (c) Find the distance from the origin to the plane $2x + y - z = 4$.
   As in problem 5d, we know that $D = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$
   In our case, $P(0, 0, 0)$, $a = 2$, $b = 1$, $c = -1$, and $d = -4$, so we have $D = \frac{|-4|}{\sqrt{4+1+1}} = \frac{4}{\sqrt{6}}$.

7. Find both the parametric and symmetric equations for the line through the points $P(3, 5, 7)$ and $Q(-6, 2, 1)$.
   To find parametric equations, We find $\overrightarrow{PQ} = \langle -9, -3, -6 \rangle$. A slightly nicer vector in the opposite direction is: $\langle 3, 1, 2 \rangle$.
   From this, using P, $\ell : \begin{cases} x = 3 + 3t \\ y = 5 + t \\ z = 7 + 2t \end{cases} \quad t \in \mathbb{R}$
   To put this in symmetric form, we solve each equation for t, yielding: $\dfrac{x-3}{3} = y - 5 = \dfrac{z-7}{2}$

8. Find an equation of the plane through the point $(1, 4, -5)$ and parallel to the plane defined by $2x - 5y + 7z = 12$.
   Since the planes are parallel, we use the same normal vector: $\vec{n} = \langle 2, -5, 7 \rangle$.
   Therefore, using $P(1, 4, -5)$, we have a plane with equation: $2(x - 1) + -5(y - 4) + 7(z + 5) = 0$.

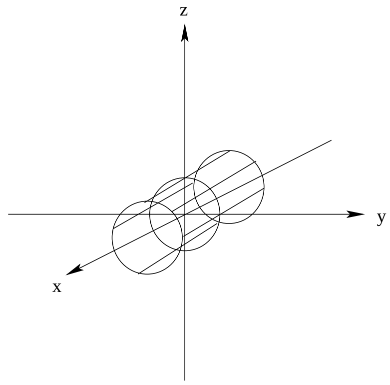
9. Sketch at least 3 traces, then sketch and identify the surface given by each of the following equations:
   Note: In the interest of getting these posted in a timely fashion, I am not including the graphs of the traces, although you should include these on your exam solutions.

   (a) $y^2 + z^2 - x = 0$
   Traces:
   If $x = 0$, we have $y^2 + z^2 = 0$, a point.
   If $x = k$, we have $y^2 + z^2 = k$, which is a circle if $k > 0$, and is empty if $k < 0$.
   If $y = 0$, we have $z^2 - x = 0$, or $x = z^2$, a parabola.
   If $z = 0$, we have $y^2 - x = 0$ or $x = y^2$, a parabola.
   Therefore, the figure is a paraboloid:



(b) $y^2 + z^2 = 1$

This is a circular cylinder sitting over the unit circle in the yz-plane:



(c) $4x^2 + 4y^2 - 2z^2 = 4$.

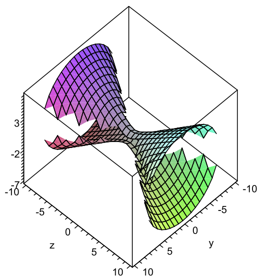
Traces:

If $x = 0$, we have $4y^2 - 2z^2 = 4$, or $y^2 - \frac{z^2}{2} = 1$ a hyperbola.

If $y = 0$, we have $4x^2 - 2z^2 = 4$, or $x^2 - \frac{z^2}{2} = 1$, a hyperbola.

If $z = 0$, we have $4x^2 + 4y^2 = 4$ or $x^2 + y^2 = 1$, a circle.

Therefore, the figure is a hyperboloid of 1 sheet:



(d) $4x^2 + y^2 - z^2 = 0$

Traces:

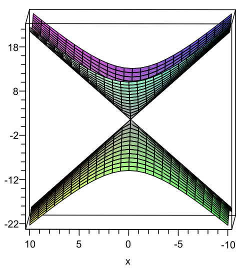
If $x = 0$, we have $y^2 - z^2 = 0$, or $y^2 = z^2$, so we have $y = \pm z$, a pair of lines.

If $y = 0$, we have $4x^2 - z^2 = 0$, or $4x^2 = z^2$, so we have $z = \pm 2x$, a pair of lines.

If $z = 0$, we have $4x^2 + y^2 = 0$, a point.

If $z = k$, we have $4x^2 + y^2 = k^2$, an ellipse.

Therefore, the figure is a cone:



(e) $y^2 - 4 = 4x^2 + z^2$

Traces:

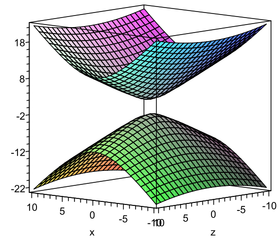
If $x = 0$, we have $y^2 - 4 = z^2$, or $y^2 - z^2 =$, a hyperbola.

If $y = 0$, we have $4x^2 + z^2 = -4$, which is empty.

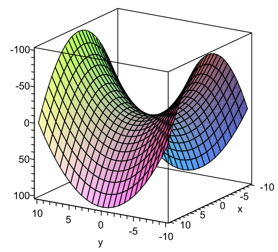
If $y = k$, we have $y^2 - 4 = 4x^2 + z^2$, which is a point if $|y| = 2$ and is an ellipse if $|y| > 2$

If $z = 0$, we have $y^2 - 4 = 4x^2$, or $y^2 - 4x^2 = 4$, a hyperbola.

Therefore, the figure is a hyperboloid of 2 sheets:



(f) $x^2 - z = y^2$
Traces:
If $x = 0$, we have $-z = y^2$, or $z = -y^2$, a parabola opening downward.
If $y = 0$, we have $x^2 - z = 0$, or $z = x^2$, a parabola opening upward.
If $z = 0$, we have $x^2 = y^2$, or $x = \pm z$, a pair of lines.
If $z = k$, we have $x^2 - y^2 = k$, which are hyperbolas whose vertices depend on the sign of k.
Therefore, the figure is a hyperbolic paraboloid (or "saddle"):



10. Let $\vec{r}(t) = \langle t^2, 4+3t, 4-3t\rangle$ be a vector valued function. Calculate the following:

(a) $\vec{r'}(t) = \langle 2t, 3, -3\rangle$

(b) $\vec{r''}(t) = \langle 2, 0, 0\rangle$

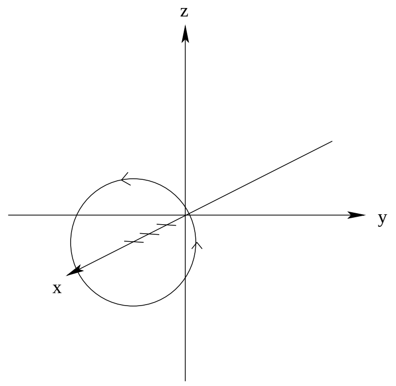
(c) $\int_0^1 \vec{r}(t)dt = \langle \int_0^1 t^2\ dt, \int_0^1 4+3t\ dt, \int_0^1 4-3t\ dy\rangle = \langle \frac{1}{3}t^3, 4t + \frac{3}{2}t^2, 4t - \frac{3}{2}t\rangle\Big|_0^1 = \langle \frac{1}{3}, \frac{11}{2}, \frac{5}{2}\rangle$

(d) the arc length of $\vec{r}(t)$ for $0 \le t \le 2$. (Just set up the integral, you do not need to evaluate it).
$L = \int_0^2 \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}\ dt$
$= \int_0^2 \sqrt{(2t)^2 + 3^2 + 3^2}\ dt = \int_0^2 \sqrt{4t^2 + 18}\ dt$

(e) Find the values of t for which $\vec{r}(t)$ and $\vec{r'}(t)$ are perpendicular.
We need $\vec{r}(t) \cdot \vec{r}'(t) = 0$. That is, $\langle t^2, 4+3t, 4-3t\rangle \cdot \langle 2t, 3, -3\rangle = (t^2)(2t) + (4+3t)(3) + (4-3t)(-3) = 2t^3 + 12 + 9t - 12 + 9t = 2t^3 + 18t = 0$
Then $2t(t^2 + 9) = 0$, so either $t = 0$, or $t^2 = -9$, which is impossible.
Therefore, $t = 0$ is the only solution.

11. Let $\vec{r}(t) = \langle 3, 4cos(t), 4sin(t)\rangle$.

(a) Sketch the curve traced out by the vector-valued function $\vec{r}(t)$. Indicate the orientation of the curve. What is geometric shape of this curve?
Using algebra, $\frac{y}{4} = \cos t$ and $\frac{z}{4} = \sin t$, so $\frac{y^2}{16} + \frac{z^2}{16} = 1$. Therefore, the curve is a circle or radius 4 in the plane $x = 3$. (See graph)



(b) Find an equation in terms of t for $s(t)$, the arc length of the curve traced out by $\vec{r}(t)$ as a function of t.

$$s(t) = \int_0^t \sqrt{0^2 + 16\cos^2 x + 16\sin^2 x}\ dx = \int_0^t \sqrt{16(\cos^2 x + \sin^2 x)}\ dx = \int_0^t \sqrt{16}\ dx = \int_0^t 4\ dx = 4x\Big|_0^t = 4t$$

(c) Find $\vec{r'}(t)$.

$\vec{r'}(t) = \langle 0, -4\sin t, 4\cos t\rangle$

(d) Find an expression for the angle between the vectors $\vec{r}(t)$ and $\vec{r'}(t)$.

Recall that $\cos\theta = \dfrac{\vec{r}(t)\cdot\vec{r'}(t)}{\|\vec{r}(t)\|\|\vec{r'}(t)\|} = \dfrac{0 - 16\sin t\cos t + 16\cos t\sin t}{\sqrt{9 + 16\cos^2 t 16\sin^2 t}\sqrt{0 + 16\sin^2 + 16\cos^2 t}} = \dfrac{0}{5+4} = 0$

Therefore, $\theta = \arccos(0) = \frac{\pi}{2}$ or $90°$ (hence they are perpendicular at all times).

(e) Find the force acting on a 5 kilogram object moving along the path given by $\vec{r}(t)$, in units of meters and seconds (assume that no other forces are acting on this mass).

Recall that $\vec{F} = m\vec{a}$. Also, $\vec{a}(t) = \vec{r}''(t) = \langle 0, -4\cos t, -4\sin t\rangle$.

Thus $\vec{F}(t) = 25\langle 0, -4\cos t, -4\sin t\rangle = \langle 0, -100\cos t, -100\sin t\rangle$ (in Newtons).

(f) Find and draw the position and tangent vectors when $t = \pi$. Find 2 different vectors that are normal to $\mathbf{r}(\pi)$ and $\mathbf{r}'(\pi)$. Draw these vectors (carefully labeled) on the same axes (as the position and tangent vectors).

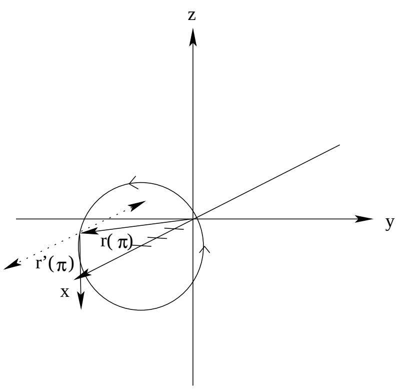
$r(\pi) = \langle 3, -4, 0\rangle$ and $r'(\pi) = \langle 0, 0, -4\rangle$

To find vectors normal to both of these, we take the cross product:

$$\vec{r}(\pi)\times\vec{r}'(\pi) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = \vec{i}(-16+0) - \vec{j}(12+0) + \vec{k}(0+0) = -16\vec{i} - 12\vec{j} + 0\vec{k}$$

Another vector normal to both is: $16\vec{i} + 12\vec{j} + 0\vec{k}$

(the dotted line vectors represent the pair of vectors normal to both $\vec{r}(\pi)$ and $\vec{r}'(\pi)$)



12. Let $\vec{v}(t) = \langle t^2 - 2t, 4t - 3, t^3\rangle$.

(a) Find $\vec{a}(t)$

$\vec{a}(t) = \langle 2t - 2, 4, 3t^2\rangle$

(b) If $\vec{r}(0) = \langle 4, -1, 0 \rangle$, find $\vec{r}(t)$.

$\vec{r}(t) = \displaystyle\int \vec{r}(t)\ dt = \langle \frac{1}{3}t^3 - t^2, 2t^2 - 3t, \frac{1}{4}t^4 \rangle + \vec{C}$

Since $\vec{r}(0) = \langle 4, -1, 0 \rangle$, $\vec{C} = \langle 4, -1, 0 \rangle$ and hence $\vec{r}(t) = \langle \frac{1}{3}t^3 - t^2 + 4, 2t^2 - 3t - 1, \frac{1}{4}t^4 \rangle$

(c) Find the force acting on an object of mass 50kg with position function $\vec{r}(t)$ (in units of meters per second).

$\vec{F}(t) = m\vec{a}(t) = 50\langle 2t - 2, 4, 3t^2 \rangle = \langle 100t - 100, 200, 150t^2 \rangle$ (in Newtons).

(d) Find the speed of the object at time $t = 2$.

Since $\vec{v}(2) = \langle 4 - 4, 8 - 3, 8 \rangle = \langle 0, 5, 8 \rangle$, $\|\vec{v}(2)\| = \sqrt{0 + 25 + 64} = \sqrt{89}$.

13. Suppose that a projectile is launched with initial velocity $v_0 = 100$ ft/s from a height of 0 feet and at an angle of $\theta = \frac{\pi}{6}$.

(a) Assuming that the only force acting on the object is gravity, find the maximum altitude, horizontal range, and speed at impact of this projectile.

Recall that the downward force of gravity is $32ft/s^2$. Since the only forces acting on this projectile are gravity and its initial velocity, we can model this situation in 2 dimensions. We take $\vec{j}$ to be the unit vector in the upward direction.

Then $\vec{a}(t) = -32\vec{j}$, so, integrating: $\vec{v}(t) = 0\vec{i} - -32\vec{j} + \vec{C}$.

Now, using trigonometry, $\vec{v}(0) = 100\cos\frac{\pi}{6}\vec{i} + 100\sin\frac{\pi}{6}\vec{j} = 50\sqrt{3}\vec{i} + 50\vec{j}$. Thus $\vec{v}(t) = 50\sqrt{3}\vec{i} + (50 - 32t)\vec{j}$.

Since $\vec{r}(0) = \vec{0}$, $\vec{r}(t) = 50\sqrt{3}t\vec{i} + (50t - 16t^2)\vec{j}$.

The maximum altitude occurs when the vertical component of $\vec{v}(t)$ is zero, that is, when $50 - 32t = 0$, or when $t = \frac{50}{32} = \frac{25}{16}$.

Therefore, the maximum altitude is found by looking at the vertical position at that time:

$50\left(\frac{25}{16}\right) - 16\left(\frac{25}{16}\right)^2 = 39.0625$ feet.

The horizontal range is found by first finding the time when the projectile hits the ground, and then finding the horizontal component of position at that time:

$50t - 16t^2 = 0$ when $t = 0$ or $50 - 16t = 0$. That is, when $t = \frac{50}{16} = \frac{25}{8}$.

Then the range is: $50\left(\frac{25}{8}\right) - 16\left(\frac{25}{8}\right)^2 \approx 270.633$ feet.

The speed at impact is found by looking at the magnitude of the velocity vector at the time of impact:

$\|\vec{v}\left(\frac{25}{8}\right)\| = \sqrt{\left(50\sqrt{3}\right)^2 + \left(50 - 32\left(\frac{25}{8}\right)\right)^2} = \sqrt{7500 + 2500} = 100 feet/sec.$

(b) Find the landing point of this projectile if it weighs 1 pound, is launched due east, and there is a southerly wind force of 4 pounds.

Because of the wind, we must move to a 3D coordinate system (let the positive x-axis be East and positive z be Up). I worded this a bit clumsily, so just to clarify, my intention is that the wind is gusting as that its force is 4 times that of gravity (I know, this is not very realistic...)

$\vec{w} = -4(32)\vec{j}$ and $\vec{g} = -1(32)\vec{k}$, so $\vec{a}(t) = 0\vec{i} - 128\vec{j} - 32\vec{k}$.

$\vec{v}(0) = 100\cos\frac{\pi}{6}\vec{i} + 0\vec{j} + 100\sin\frac{\pi}{6}\vec{k} = 50\sqrt{3}\vec{i} + 0\vec{j} + 50\vec{k}$.

Then $\vec{v}(t) = (0\vec{i} - 128t\vec{j} - 32t\vec{k}) + (50\sqrt{3}\vec{i} + 0\vec{j} + 50\vec{k}) = 50\sqrt{3}\vec{i} - 128t\vec{j} + 50 - 32t\vec{k}$

As before, the projectile hits the ground after $t = \frac{25}{8}$ seconds.

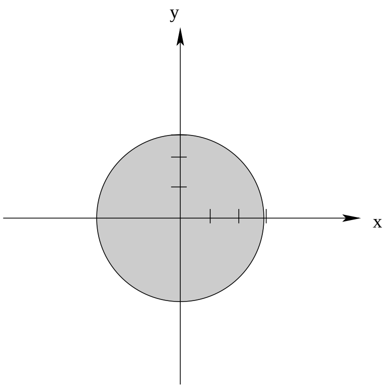
Notice that we set up our coordinate system so that $\vec{r}(0) = \vec{0}$. Therefore, $\vec{r}(t) = 50\sqrt{3}t\vec{i} - 64t^2\vec{j} + (50t - 16t^2)\vec{k}$.

Thus the final position of the projectile is: $\vec{r}(\frac{25}{8}) = \langle 50\sqrt{3}\left(\frac{25}{8}\right), -64\left(\frac{25}{8}\right)^2, 50\left(\frac{25}{8}\right) - 16\left(\frac{25}{8}\right)^2 \rangle \approx \langle 270.63, -625, 0 \rangle$ where the coordinates are in feet.

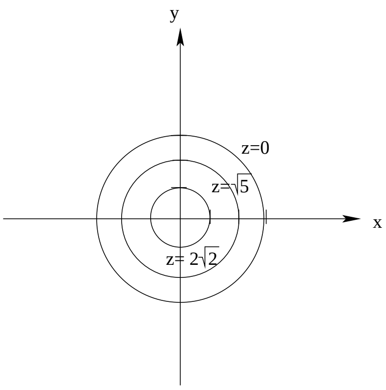
14. Let $f(x, y) = \sqrt{9 - x^2 - y^2}$.

(a) Sketch the domain of f in the x, y-plane.

We need $9 - x^2 - y^2 \geq 0$, or $9 \geq x^2 + y^2$, so the domain is the set off all points on or inside the circle of radius 3 centered at the origin.

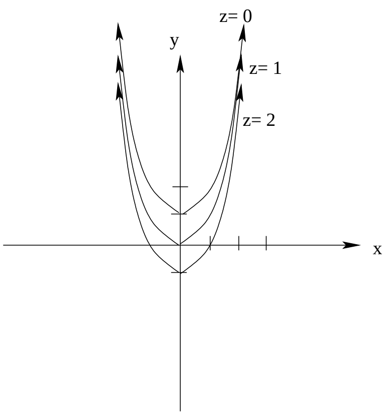


(b) Graph contours for $z = f(x, y)$ for $z = 0, \sqrt{5}$,and $2\sqrt{2}$.
Contours:
If $z = 0 = \sqrt{9 - x^2 - y^2}$, then $0 = 9 - x^2 - y^2$, or $x^2 + y^2 = 9$.
If $z = sqrt5 = \sqrt{9 - x^2 - y^2}$, then $5 = 9 - x^2 - y^2$, or $x^2 + y^2 = 4$.
If $z = 2\sqrt{2} = \sqrt{9 - x^2 - y^2}$, then $8 = 9 - x^2 - y^2$, or $x^2 + y^2 = 1$.



15. Given the function $z = f(x, y) = 1 + x^2 - y$:

(a) Sketch contours for this function for $z = 0, 1, 2$
If $z = 0 = 1 + x^2 - y$, then $y = x^2 + 1$
If $z = 1 = 1 + x^2 - y$, then $y = x^2$
If $z = 2 = 1 + x^2 - y$, then $y = x^2 - 1$
The contours are all parabolas.



(b) What type of curves are the x-cross sections and the y-cross sections of f?
If $x = k$, then $z = 1 + k^2 - y$, which is a line of slope -1.
If $y = k$, then $z = 1 + x^2 - k$, which is a parabola.