Math 323
Exam 4 - Practice Problems

1. Cascade Container Company produces a cardboard shipping crate at three different plants in amounts x, y, and z, respectively. Their annual revenue is $R(x, y, z) = 2xyz^2$. The company needs to produce 1000 crates annually. How many crates should they produce at each plant in order to maximize their revenue?

2. Use Lagrange multipliers to maximize $f(x, y) = 4x^2y$ subject to the constraint $x^2 + y^2 = 3$.

3. Evaluate $\iint_R (y + x)\, dA$, where R is the region bounded by $x = 0$, $y = 0$, and $2x + y = 4$.

4. For each of the double integrals given below, first graph the region of integration. Then reverse the order of integration for the iterated integral and then evaluate the integral exactly.

   (a) $\int_0^1 \int_y^1 3xe^{x^3}\, dx\, dy$

   (b) $\int_0^1 \int_{\sqrt{x}}^1 \frac{3}{4 + y^3}\, dy\, dx$

   (c) $\int_0^\pi \int_y^\pi \left(1 - \frac{\cos x}{x}\right)\, dx\, dy$

5. Express the volume of the solid bounded by the curves $z = x + 2, z = 0, x = y^2 - 2$ and $x = y$ as a double integral in rectangular coordinates. Also, sketch the region in the plane for the integration. DO NOT EVALUATE THE INTEGRAL.

6. Evaluate $\iint_R x\, dA$ where R is the region in the polar plane bounded by $r = 1 - \sin\theta$.

7. Calculate the mass of a lamina that occupies the plane region R bounded by the curve $(x - 1)^2 + y^2 = 1$ with density function $\rho(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$.

8. Sketch the region of integration for $\int_{\pi/4}^{3\pi/4} \int_1^{2/\sin\theta} r\, dr\, d\theta$.

9. Find the surface area of the surface S where S is first octant portion of the hyperbolic paraboloid $z = x^2 - y^2$ that is inside the cylinder $x^2 + y^2 = 1$.

10. Let $I = \int_0^2 \int_{x^2}^4 \int_0^{4-y} x + yz\, dz\, dy\, dx$. Sketch the solid Q over which the iterated integral takes place, and rewrite the iterated integral in the order $dx\, dz\, dy$. DO NOT EVALUATE THE INTEGRAL.

11. Find the mass of the cylinder of radius 3 between $z = 0$ and $z = 4$ if the density at the point (x, y, z) is given by $\delta(x, y, z) = z + \sqrt{x^2 + y^2}$.

12. Let Q be the region between $z = (x^2 + y^2)^{3/2}$ and $z = -1$, and inside $x^2 + y^2 = 4$. Sketch the region Q, and then write $\iiint_Q \sqrt{x^2 + y^2}e^z\, dV$ as an integral in the best(for this example) 3-dimensional coordinate system. DO NOT EVALUATE THE INTEGRAL.

13. Set up a triple integral in spherical coordinates for the volume V of the region between $z = \sqrt{3x^2 + 3y^2}$ and the sphere $x^2 + y^2 + z^2 = 16$. Be sure to include a sketch of the region, and DO NOT EVALUATE THE INTEGRAL.

14. (a) Let Q be the region between $z = \sqrt{3x^2 + 3y^2}$ and $z = \sqrt{4 - x^2 - y^2}$. Sketch the region Q

    (b) Set up, but do not evaluate a triple integral in rectangular coordinates that gives the volume of Q.

    (c) Set up, but do not evaluate a triple integral in cylindrical coordinates that gives the volume of Q.

    (d) Set up, but do not evaluate a triple integral in spherical coordinates that gives the volume of Q.

    (e) Pick one of the triple integrals you found above and evaluate it in order to find the volume of Q exactly.