

1. (Modified from the 2007 AP Calculus AB exam) Let  $R$  be the region in the first and second quadrants that is bounded above by the graph of  $y = \frac{20}{1+x^2}$  and below by the horizontal line  $y = 2$ . Let  $S$  be the region in the first quadrant bounded above by  $y = \frac{20}{1+x^2}$  and below by the  $x$ -axis. (This problem continues on the following pages.)

(a) Evaluate  $\int \frac{1}{(1+x^2)^2} dx$

(b) Evaluate  $\int_0^3 \frac{1}{(1+x^2)^2} dx$ .

(c) Evaluate  $\int_0^\infty \frac{1}{(1+x^2)^2} dx$ .

- (d) Find the volume of the solid generated when  $R$  is rotated about the  $x$ -axis.

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- (e) Find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.
- (f) Find the volume of the solid generated when  $S$  is rotated about the  $x$ -axis.
- (g) Find the volume of the solid generated when  $S$  is rotated about the  $y$ -axis.
- (h) Setup, but do not evaluate, an integral expression that gives the length of the boundary curve for the region  $R$ .
- (i) Setup, but do not evaluate, an integral expression that gives the surface area of the solid that is formed when the region  $R$  is rotated about the line  $y = 2$ .

- (j) The region  $R$  is the base of a solid. For this solid, the cross-sections perpendicular to the  $x$ -axis are semicircles. Find the volume of this solid.

2. A person standing on a platform 20 ft above the ground is holding one end of a chain. On the other end of the chain is a 10 lb weight. If the chain weighs 2 lbs/ft. How much work is required to lift the weight to the top of the platform?

3. Compute the following integrals.

(a)  $\int 5^x dx$

(c)  $\int xe^x dx$

(b)  $\int \frac{x}{1-x^2} dx$

(d)  $\int \frac{1}{x^2-5x+6} dx$

(e)  $\int \frac{1}{(x^2 - 6x + 34)^{3/2}} dx$

(g)  $\int \tan^3 x \sec^5 x dx$

(h)  $\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}$

(f)  $\int \ln x dx$

4. Determine whether the following sequences converge or diverge. For those that converge, find the limit.

(a)  $\left\{ 1 + \frac{(-1)^n}{n} \right\}$

(b)  $\left\{ \frac{4^n - 7}{9^n} \right\}$

5. Find the sum of each series:

(a)  $\sum_{n=2}^{\infty} e^{-n}$

(b)  $\sum_{n=1}^{\infty} \frac{1}{(2n-3)(2n-1)}$

6. Determine whether the following series converge or diverge. Make sure to show all work leading to your conclusion.

$$(a) \sum_{n=1}^{\infty} \frac{3n+5}{n^2+7}$$

$$(b) \sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

$$(c) \sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$$

7. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Make sure to show all work leading to your conclusion.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$$

$$(b) \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{3 + \ln n}$$

$$(c) \sum_{n=1}^{\infty} (-1)^n \frac{3n^2}{n^3+1}$$

8. For each of the following power series, find the interval of convergence and the radius of convergence:

(a) 
$$\sum_{n=0}^{\infty} \frac{1}{n3^n} (x+4)^n$$

Interval of Convergence: \_\_\_\_\_

Radius of Convergence: \_\_\_\_\_

(b) 
$$\sum_{n=0}^{\infty} \frac{1}{n^n} x^n$$

Interval of Convergence: \_\_\_\_\_

Radius of Convergence: \_\_\_\_\_

9. Find a power series in  $x$  that has the given function as its sum. Also find the interval of convergence.

(a)  $\frac{1}{1+x^3}$

(b)  $\sin \frac{2x}{3}$

10. Find the first four terms of the Taylor series for the function  $f(x) = \frac{1}{1-x}$  centered at  $c = 2$ .

11. Approximate the integral  $\int_0^1 x \sin(x^3) dx$  to six decimal places.