

1. Find each limit, (if it exists).

$$(a) \lim_{x \rightarrow 0^+} \frac{\ln x}{\csc x}$$

$$(b) \lim_{x \rightarrow 1} \frac{2x^2 - (3x + 1)\sqrt{x} + 2}{x - 1}$$

$$(c) \lim_{x \rightarrow \infty} \frac{\sqrt{9x + 1}}{\sqrt{x + 1}}$$

$$(d) \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

$$(e) \lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos x}$$

$$(f) \lim_{x \rightarrow \infty} \frac{2x^2 - 1}{5x^2 + 3x}$$

$$(g) \lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 3}$$

$$(h) \lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$$

$$(i) \lim_{x \rightarrow 0} x \cot x$$

$$(j) \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$

$$(k) \lim_{x \rightarrow 0^+} (1 + ax)^{\frac{b}{x}}$$

$$(l) \lim_{x \rightarrow 0^+} x^x$$

2. Find a value of c that makes the function

$$f(x) = \begin{cases} \frac{9x-3\sin 3x}{5x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

continuous at $x = 0$

3. (a) The **sine integral** $Si(x) = \int_0^x \frac{\sin u}{u} du$ is a useful function in applied mathematics.

$$\text{Find } \lim_{x \rightarrow 0} \frac{Si(x)}{x}$$

(b) The **Fresnel cosine integral** $C(x) = \int_0^x \cos(u^2) du$ is used in the analysis of the diffraction of light.

$$\text{Find: } \lim_{x \rightarrow 0} \frac{C(x) - x}{x^5}$$

4. If air resistance is disregarded, the acceleration of a falling object is given by $a(t) = g$, where g is a gravitational constant, so the velocity of an object t seconds after being released is given by: $v(t) = gt$.

If we assume air resistance is proportional to the velocity v , it can be shown that the velocity of an object t seconds after being released is given by: $v(t) = \frac{mg}{k} (1 - e^{-kt/m})$, where m is the mass of the object and k is a constant air-resistance coefficient. In the following problems, we will explore the consequences of taking air resistance into account when studying the velocity, $v(t)$ of a falling object.

(a) Find $\lim_{k \rightarrow 0^+} v(t)$

[Consequently, $k = 0$ corresponds to the case of no air resistance.]

(b) Find $\lim_{t \rightarrow \infty} v(t)$

[Consequently, a body's velocity tends to a finite limit, its *terminal velocity*, after it has fallen a sufficiently long time.]

(c) Find $\lim_{m \rightarrow 0^+} v(t)$

[Consequently, a light "feathery" body falls very slowly through the air.]

(d) Find $\lim_{m \rightarrow \infty} v(t)$

[Consequently, a heavy body tends to fall much as it would with no air resistance.]