1. Determine whether the following improper integrals converge or diverge. For those that do converge, find their value.

(a) $\displaystyle\int_2^\infty \frac{1}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_1^5 \frac{1}{\sqrt{x-1}}\,dx$

(b) $\displaystyle\int_2^\infty \frac{1}{x\ln x}\,dx$

(e) $\displaystyle\int_{-1}^0 \frac{1}{\sqrt[5]{x}}\,dx$

(c) $\displaystyle\int_0^\infty \frac{x}{e^x}\,dx$

(f) $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$

2. Use comparisons to determine whether the following improper integrals converge or diverge:

(a) $\displaystyle\int_1^\infty \frac{2+\cos x}{x^2}\,dx$

(b) $\displaystyle\int_1^\infty \frac{1}{\sqrt[3]{x^3-x}}\,dx$

(c) $\displaystyle\int_1^\infty \frac{1}{x^{1.1}+x+2}\,dx$

(d) $\displaystyle\int_1^\infty \frac{1}{x^{0.9}+x+2}\,dx$

3. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{1+x}{1+x^2}\,dx$ diverges, but that $\displaystyle\lim_{t\to\infty}\int_{-t}^{t} \frac{1+x}{1+x^2}\,dx = \pi$

4. A mechanic decides to make a funnel by rotating the curve $y = \dfrac{1}{x}$ from $x = 1$ to $x = H$ about the x-axis.

(a) Find the volume of the funnel, in terms of H.

(b) Set up (but DO NOT evalutate) an integral that represents the surface area of the funnel in terms of H.

(c) If we let $H \to \infty$, find the volume of the resulting "infinitely long" funnel (often called "Gabriel's Horn")

(d) Use a comparison to show that if we let $H \to \infty$, the resulting "infinitely long" funnel has infinite surface area.