1. In each of the following problems, a formula for the *n*th term a_n of a sequence $\{a_n\}$ is given. Find the first *four* terms: a_1, a_2, a_3 , and a_4 of each sequence and find $\lim_{n \to \infty} a_n$ if it exists.

(a)
$$a_n = \frac{3}{n!}$$

(b)
$$a_n = \frac{4 - n^2}{3 + n^2}$$

(c)
$$a_n = (-1)^n \frac{n}{n+1}$$

(d)
$$a_n = 7$$

- 2. In each of the following problems, a sequence $\{a_n\}$ is defined recursively. Find the first *five* terms: a_1, a_2, a_3, a_4 and a_5 of each sequence. Then, using a calculator is necessary, give an educated guess for $\lim_{n\to\infty} a_n$, if it exists.
 - (a) $a_1 = 1$ $a_{n+1} = a_n + \frac{1}{2^n}$ for $n \ge 1$

(b)
$$a_1 = \sqrt{6}$$
 $a_{n+1} = \sqrt{6 + a_n}$ for $n \ge 1$

(c)
$$a_1 = 1$$
 $a_2 = 1$ $a_{n+2} = a_{n+1} + a_n$ for $n \ge 1$

3. Determine whether each sequence converges or diverges. For those that converge, find the limit.

(a)
$$\left\{ \frac{1-5n^4}{n^4+7n^3} \right\}$$

(b)
$$\left\{ \left(1 + \frac{7}{n}\right)^n \right\}$$

(c)
$$\left\{\frac{2^n+5}{3^n}\right\}$$

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(d) <	$\left\{\sec^{-1}n\right\}$				

(e)
$$\left\{\frac{\sin n}{n^2}\right\}$$

(f)
$$\{\ln(2n+1) - \ln n\}$$

(g)
$$\left\{\frac{n!}{n^n}\right\}$$
 [Hint: Compare with $\frac{1}{n}$].

Name:

4. The so-called **hailstone** sequence is defined by:

$$a_{n+1} = \begin{cases} 3a_n + 1 & \text{if } a_n \text{ is odd} \\ \frac{1}{2}a_n & \text{if } a_n \text{ is even} \end{cases}$$

(a) Let $a_1 = 17$. Write down the terms of the sequence until you reach 1.

(b) Let $a_1 = 30$. Write down the terms of the sequence until you reach 1.

(c) Start with some other positive integer value for a_1 and write down the terms of the sequence until you reach 1.

(d) Start with yet another positive integer value for a_1 and write down the terms of the sequence until you reach 1.

(e) Either show that you always reach 1 no matter which positive integer value for a₁ you start with or find an example of a sequence that you can prove never reaches 1.
(Attach additional work if needed)
[Note: If you are successful in solving this problem, you will be the first person in history to have solved this problem. No solution is known.]