

1. Let  $f(x) = -\ln(1-x)$ .

(a) Find  $f'(x)$ ,  $f''(x)$ ,  $f'''(x)$ ,  $f^{(4)}(x)$ , and  $f^{(n)}(x)$

(b) Find  $f'(0)$ ,  $f''(0)$ ,  $f'''(0)$ ,  $f^{(4)}(0)$ , and  $f^{(n)}(0)$

(c) Find the  $n$ th-degree Maclaurin polynomial of  $f$

(d) Find the Maclaurin series for  $f(x)$ . (Do not verify that  $\lim_{n \rightarrow \infty} R_n(x) = 0$ )

(e) Find the interval of convergence for this Maclaurin series.

2. Find the first five terms of the Taylor series for the following functions at a given value  $c$ :

(a)  $f(x) = \cos x$  at  $c = \frac{\pi}{3}$

(b)  $f(x) = \sqrt{x}$  at  $c = 4$

3. Find the Maclaurin series for  $f(x) = 6x^4 - 2x^3 + 4x^2 + x + 7$ . Show all work!

4. Find the sum of each of the following infinite series. Give exact answers.

(a)  $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots + (-1)^n \frac{1}{(2n)!} + \cdots$

(b)  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^n \frac{1}{n+1} + \cdots$

(c)  $\frac{\pi}{6} - \frac{\pi^3}{6^3 \cdot 3!} + \frac{\pi^5}{6^5 \cdot 5!} - \cdots + (-1)^n \frac{\pi^{2n+1}}{6^{2n+1} (2n+1)!} + \cdots$

5. Find the Indicated Taylor polynomial and remainder given  $f$ ,  $c$  and  $n$ :

(a)  $f(x) = \cos x$ ,  $c = \frac{\pi}{4}$ ,  $n = 4$

(b)  $f(x) = \frac{1}{(x-3)^2}$ ,  $c = 4$ ,  $n = 5$

6. If we used problem 6(a) to approximate  $\cos 47^\circ$ , what decimal place accuracy could we achieve?

7. (a) Use problem 6(b) to approximate  $\frac{1}{(.9)^2}$

(b) Use the Taylor remainder in problem 6(b) to estimate the error in this approximation.

8. Determine the number of decimal places of accuracy that the given approximation formula yields for  $|x| \leq 0.1$ .

(a)  $e^x \approx 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$

(b)  $\sin x \approx x - \frac{1}{6}x^3 + \frac{1}{120}x^5$

(c)  $\ln(1+x) \approx x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$