

1. Find the arc length of the graph of the following equations from A to B .

(a) $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$; $A(0, \frac{2}{3}\sqrt{2})$ $B(3, \frac{11}{3}\sqrt{11})$

(b) $x = \frac{y^3}{3} + \frac{1}{4y}$; $A(\frac{7}{12}, 1)$ $B(\frac{109}{12}, 3)$

(c) $y = \int_0^x \sqrt{\sec^4 t - 1} dt$; $A(-\frac{\pi}{4}, -0.54\dots)$ $B(\frac{\pi}{4}, 0.54\dots)$

2. The graph of the equation $y = x^3$ from $A(0, 0)$ to $B(2, 8)$ is revolved about the x -axis. Find the area of the resulting surface.

3. The graph of the equation $y = x^2$ from $A(0, 0)$ to $B(\sqrt{2}, 2)$ is revolved about the y -axis. Find the area of the resulting surface.

4. Set up the integral that will solve the following problem: A manufacturer needs to make corrugated metal sheets 36 inches wide with cross sections in the shape of the curve $y = \frac{1}{2} \sin \pi x$, $0 \leq x \leq 36$. How wide must the original flat sheets be for the manufacturer to produce these corrugated sheets? [You **do not** need to evaluate this integral]

5. Derive a formula for the surface area of a sphere of radius r .

6. An astroid is a curve with an equation of the form $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, for some fixed positive number a . An astroid is traced by a point on a circle of radius $\frac{a}{4}$ that is rolled on the inside of a circle of radius a . It was first studied by Johann Bernoulli in 1691.

(a) Find the total length of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

(b) Find the area of the surface generated by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x -axis.

(c) Look carefully at the integrals that you evaluated in the two parts above. Is there a problem with either of these integrals? Can you think of a way to avoid this difficulty? We will learn more about this problem in chapter 10.