Definitions

Euclid's *Elements of Geometry* translation by Thomas L. Heath.

A *point* is that which has no part.
A *line* is breadthless length.
A *straight line* is a line which lies evenly with the points on itself.
A *surface* is that which has length and breadth only.
A *plane surface* is a surface which lies evenly with the straight lines on itself.

Definitions of some terms used in the incorrect proof of "All triangles are isosceles triangles."

Triangle

1. (Euclid's) *Rectilineal figures* are those which are contained by straight lines, *trilateral* figures being those contained by three.
2. A *triangle* is the union of three noncollinear points and the three segments defined by those three points. Each of the three noncollinear points is called a *vertex* of the triangle. Each of the three segments is called a *side* of the triangle.
3. If $\{A, B, C\}$ are noncollinear points, then the *triangle* ABC is the set
$$\triangle ABC = \overline{AB} \cup \overline{BC} \cup \overline{CA}.$$
Each noncollinear point $A,\ B,\ C$ is called a *vertex* of $\triangle ABC$. Each segment $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ is called a *side* of $\triangle ABC$.

Collinear and *Noncollinear*

1. A set of points is *noncollinear* if there does not exist a line which contains them.
2. Points are *collinear* if they lie on the same line. Points are *noncollinear* if they are not collinear.
3. A set of points $\mathcal{S}$ is *collinear* if there is a line l such that $\mathcal{S} \subset l$. $\mathcal{S}$ is *noncollinear* if $\mathcal{S}$ is not a collinear set.

Segment or *Line segment*

1. A *line segment* is the union of two distinct points and all points between those two points.
2. If A and B are distinct points then the *line segment* from A to B is the set
$$\overline{AB} = \{C : C = A \text{ or } C = B \text{ or } C \text{ is between } A \text{ and } B\}.$$

Betweenness of points

1. (Synthetic) a primitive or undefined term
2. (Metric) A point C is *between* A and B if A, B, C are distinct collinear points and if $AC + CB = AB$. Here, AB represents the distance from A to B or length of $\overline{AB}$.

Distance (Metric)

A *distance* function on a set $\mathcal{S}$ is a function $d : \mathcal{S} \times \mathcal{S} \to \mathbb{R}$ such that for all $A, B \in \mathcal{S}$
(1) $d(A, B) \geq 0$;
(2) $d(A, B) = 0$ if and only if $A = B$;
(3) $d(A, B) = d(B, A)$.

Isosceles triangle

1. (Euclid's) An *isosceles triangle* is that which has two sides alone equal.
2. An *isosceles triangle* is a triangle with at least two congruent sides.

Congruent segments

1. (Synthetic) *congruence* is a primitive or undefined
2. (Metric) Two segments are congruent if they have the same length.
3. (Metric) $\overline{AB} \cong \overline{CD}$ if and only if $AB = CD$.

Ray

If A and B are distinct points then the *ray* AB is the set

$$\overrightarrow{AB} = \overline{AB} \cup \{C : B \text{ is between } A \text{ and } C\}.$$

The point A is called the endpoint of the ray AB.

Angle

1. An *angle* is the union of two noncollinear rays with a common endpoint.
2. If A, B, C are noncollinear points, then the *angle ABC* is the set $\angle ABC = \overrightarrow{BA} \cup \overrightarrow{BC}$.

Bisector of an angle

The *bisector of an angle* $\angle ABC$ is a ray $\overrightarrow{BD}$ where D is in the interior of $\angle ABC$ and $\angle ABD \cong \angle DBC$.

Congruent angles

1. (Synthetic) *congruence* is a primitive or undefined
2. (Metric) Two angles are congruent if they have the same measure.
3. (Metric) $\angle ABC \cong \angle DEF$ if and only if $m\angle ABC = m\angle DEF$.

Angle measure

(Metric) *Angle measure* is a function on the set $\mathcal{A}$ of all angles such that $m : \mathcal{A} \to (0, r)$, $r > 0$, such that if $\overrightarrow{BA}$ is on the edge of a half-plane H. For every $a \in (0, r)$ there is exactly one ray $\overrightarrow{BP}$, with P in H such that $m\angle PAB = a$. (r is usually taken to be 180 or π.)

Half-plane (Will be clarified when axioms are given.)

Interior of an angle

The *interior of an angle* $\angle ABC$ is the set of all points X such that X is on the same side of $\overleftrightarrow{AB}$ as C and X is on the same side of $\overleftrightarrow{BC}$ as A.

Same side of a line

Two distinct points A and B lie on the *same side* of a line l if $\overline{AB}$ does not intersect l.

Midpoint

M is the *midpoint* of segment $\overline{AB}$, if M lies on $\overleftrightarrow{AB}$ and $\overline{AM} \cong \overline{MB}$.

Perpendicular

1. Two lines are perpendicular if their union contains four congruent angles.
2. Two lines are perpendicular if their union contains a right angle.
3. Two lines are perpendicular if their union contains two adjacent congruent angles.

Right angle

1. (Euclid's) When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is *right*. The straight line standing on the other is *perpendicular* to that on which it stands.
2. (Metric) A *right angle* is an angle that measures $\frac{1}{2}$r. (r is usually 180 or π.)
3. A *right angle* is an angle whose sides are lie on two perpendicular lines.