

Instructions: This project is designed to give you an opportunity to explore some of the concepts from set theory. Complete as much of this project as you can by 4:00pm on the due date (Friday, April 9th). You should write up your solutions neatly and all pertinent work leading up to your solution should be included as well. If you consult any references (books or online material), cite the relevant sources either in footnotes or at the end of your project.

1. Given the argument:

$$\begin{array}{l} p \vee \sim r \\ q \rightarrow r \\ \sim p \\ \hline \therefore \sim q \end{array}$$

- (a) (5 points) Use the truth table to demonstrate that this argument is valid.
- (b) (5 points) Write a two column proof that demonstrates that this argument is valid.
2. (3 points each) Use Venn diagrams to decide whether or not the following statements are true. If a statement is false, give a **specific counterexample** that shows that it cannot be true. If a statement is true, give a **specific example** that shows set equality holds.
- What I mean by a *specific example* is starting with specific sets A , B , and C in roster notation and then showing the result of applying the set operations to these specific sets.
- (a) $(A \cap B) \cup (A \cap C) = (B \cup C) \cap A$
- (b) $C - [(A - B) \cup (B - A)] = [C - (A \cup B)] \cup (A \cap B \cap C)$
3. (4 points) Draw a Venn diagram showing all 16 possible regions for four sets, A , B , C , and D . Number each region in your diagram and then make a list indicating which of the four sets is represented in each region.
4. Remember the slot machine from a previous worksheet whose first wheel has 3 cherries, 5 oranges, 2 bars, 4 bells, and 6 pears, second wheel has 5 cherries, 7 oranges, 4 bars, 1 bell, and 3 pears, and third wheel has 1 cherry, 6 oranges, 2 bars, 3 bells, and 8 pears.
- (a) (5 points) Recall that to win on this slot machine, you need to get 3 matching symbols when the wheels come to rest. Compute the probability of getting 3 Cherries, the probability of getting 3 Oranges, the probability of getting 3 Bars, the probability of getting 3 Bells, and the probability of getting 3 Pears.
- (b) (5 points) It costs \$1 for each spin on this slot machine. The payouts for this slot machine are as follows: 3 Oranges pays \$5, 3 Pears pays \$10, 3 Cherries pays \$100, 3 Bars pays \$100, and 3 Bells pays \$100. Find the expected value for playing this slot machine [Hint: What is the probability of losing?].