

## Math 102

### Exam 2: Additional Practice Problems

- For each of the following, state whether the situation is an example of inductive or deductive reasoning:
  - You notice that your houseplants seem to grow better if you water them in the morning rather than in the evening, so you decide to start watering them every morning right before you leave to go to school.
  - After hearing a debate on the radio, you decide to construct a truth table in order to determine whether or not the logical argument given by one of the participants is valid.
  - The last couple of times you have gone to the grocery store on Friday afternoon, you noticed that they were giving away free samples, so you decide to start doing your grocery shopping on Friday afternoon every week.
- Use inductive reasoning to predict the next two terms in each of the following sequences:
  - $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$
  - 1, 2, 4, 7, 11, 16, ...
  - 2, 5, 7, 12, 19, 31, ...
- Determine whether or not each of the following are statements:
  - I went to the store last night.
  - Did you remember to get milk at the store?
  - I forgot to pick up a gallon of milk at the store.
- Negate each of the following statements, then rewrite them as English sentences:
  - All bees are busy.
  - Some things are better left unsaid.
  - I got up early on Saturday and went to the gym.
  - This summer I will get a job or I will take classes.
  - If I eat my vegetables then I will get dessert
- Given  $p$ : "I studied for this exam",  $q$ : "I got a good grade on this exam",  $r$ : "I understand truth tables", and  $s$ : "I am not good at doing proofs", translate the following statements into words:
  - $p \wedge (\sim s) \rightarrow q$
  - $(\sim p \vee s) \rightarrow \sim q$
  - $p \leftrightarrow (\sim s \vee \sim r)$
  - $(p \rightarrow (r \wedge (\sim s))) \rightarrow q$
- Explain, in your own words, the difference between "exclusive or" and "inclusive or"
  - Give real world examples that illustrate both "exclusive or" and "inclusive or"
- Given the statements:  $p$ : There is a full moon tonight, and  $q$ : I will go for a walk on the beach
  - Write the conditional statement relating  $p$  to  $q$  in words.
  - Write the converse in words.
  - Write the inverse in words.
  - Write the contrapositive in words.
  - Indicate which of these statements above are logically equivalent to each other. You do not need to prove your answer.
- According to one of DeMorgan's Laws,  $\sim (p \vee q)$  is logically equivalent to  $(\sim p) \wedge (\sim q)$ . Use truth tables to prove that these two statements are logically equivalent. Then, explain in your own words why the fact that these two statements are equivalent makes sense.
- Given that  $p$  is true,  $q$  is false,  $r$  is true, and  $s$  is true:
  - What is truth value of the statement:  $\sim (p \vee q) \rightarrow (r \wedge \sim s)$
  - How many rows would the full truth table for the expression  $\sim (p \vee q) \rightarrow (r \wedge \sim s)$  have?

10. Build truth tables for the following logical statements:

- (a)  $(p \wedge (\sim q) \rightarrow q$
- (b)  $\sim q \rightarrow (p \vee \sim r)$
- (c)  $(p \rightarrow q) \leftrightarrow \sim (q \wedge r)$

11. Identify the form of the following arguments, and state whether the given argument is valid:

- (a) If I have enough money saved up, then I will go to Mexico for Spring Break. I did not go to Mexico for Spring Break. Therefore, I did not have enough money saved up.
- (b) If I lie on my tax return, then I will get audited by the IRS. I got audited by the IRS. Therefore, I lied on my tax return.
- (c) I will go to Mexico for Spring Break or I will spend Spring Break with my family. I did not spend Spring Break with my family. Therefore, I went to Mexico for Spring Break.

12. (a) Draw an Euler diagram for the statements: "Some A's are B's", "All C's are not A's", and "All D's are A's"  
 (b) State a valid conclusion that can be made based on the statements in part (a) above.

13. Use Euler diagrams to determine whether the following syllogisms are valid or invalid:

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| <p>Some exams are too long.<br/>                 (a) <math>\frac{\text{Some exams are too difficult.}}{\text{Therefore, some exams are too long and too difficult.}}</math></p> | <p>Some dogs chase cats.<br/>                 (b) <math>\frac{\text{All dogs have fleas.}}{\text{Therefore, some cat-chasing dogs have fleas.}}</math></p> |
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14. Use a truth table to determine whether or not the following argument is valid:

If I work hard, then I will get a raise.  
 If I get a raise, then I will not have to get a second job.  
 I got a second job.  


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 Therefore, I did not work hard.

15. Given the argument:

$p \rightarrow q$   
 $\sim (q \wedge r)$   
 $r$   


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 $\therefore \sim p$

Fill in the missing reasons in the following two column proof:

Statement	Reason
1. $\sim (q \wedge r)$	
2. $\sim q \vee \sim r$	
3. $r$	
4. $\sim (\sim r)$	
5. $\sim q$	
6. $p \rightarrow q$	
7. $\sim q \rightarrow \sim p$	
8. $\sim p$	

16. Write a 2-column proof to verify the following argument:

$t \rightarrow p$   
 $s \vee t$   
 $p \rightarrow q$   
 $\sim q$   


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 $\therefore s$