

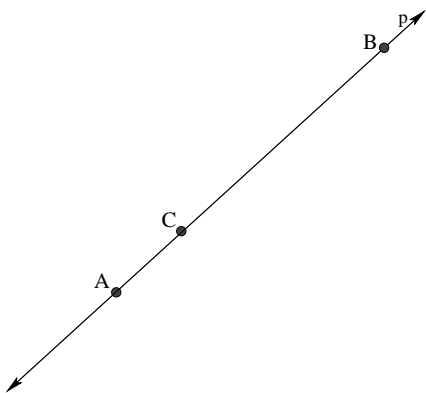
1. Give the definition of each of the following terms:
 - (a) A complete quadrangle
 - (b) A complete quadrilateral
 - (c) A perspectivity between pencils of points
 - (d) A perspectivity between pencils of lines
 - (e) A projectivity between pencils of points
 - (f) The harmonic conjugate of a point C with respect to points A and B .
 - (g) A point conic
 - (h) A line conic
2. State each of the following:
 - (a) Desargues' Theorem
 - (b) The Fundamental Theorem of Projective Geometry
3. True or False
 - (a) In a plane projective geometry, if two triangles are perspective from a point, then they are also perspective from a line.
 - (b) In the Poincaré Half Plane, if two triangles are perspective from a point, then they are also perspective from a line.
 - (c) In a plane projective geometry, if two triangles are perspective from a line, then they are also perspective from a point.
 - (d) Every point in a plane projective geometry is incident with at least 4 distinct lines.
 - (e) If $H(AB, CD)$ then $H(CD, BA)$.
 - (f) If $H(AB, CD)$ and $H(AB, C'D)$ then $C = C'$
 - (g) If A, B, C and A', B', C' are distinct elements in pencils of points with distinct axes p and p' , there there exists a perspectivity such that $ABC \stackrel{\circ}{\sim} A'B'C'$
4. Prove that Axiom 3 is independent of Axiom 1 and Axiom 2.
5.
 - (a) State and prove the dual of Axiom 3.
 - (b) State and prove the dual of Axiom 4.
6.
 - (a) Prove that a complete quadrangle exists.
 - (b) Draw a model for a complete quadrangle $EFGH$.
 - (c) Identify the pairs of opposite sides in the quadrangle $EFGH$.
 - (d) Construct and identify the diagonal points of the quadrangle $EFGH$.
7.
 - (a) Prove that a complete quadrilateral exists.
 - (b) Draw a model for a complete quadrilateral $abcd$.
 - (c) Identify the pairs of opposite points in the quadrilateral $abcd$.
 - (d) Construct and identify the diagonal lines of the quadrilateral $abcd$.
8.
 - (a) Construct an example of two triangles that are perspective from a point. Be sure to identify the point O that the triangles are perspective from.
 - (b) Are these two triangles also perspective from a line? If so, identify the line that the triangles are perspective from. If not, explain why they cannot be perspective from a line.
9. Illustrate a projectivity from a pencil of lines a, b, c with center O to a pencil of lines a', b', c' with center $O' \neq O$.

10. Prove each of the following:

- (a) The dual of Desargues' Theorem
- (b) Theorem 4.6
- (c) The Fundamental Theorem of Projective Geometry

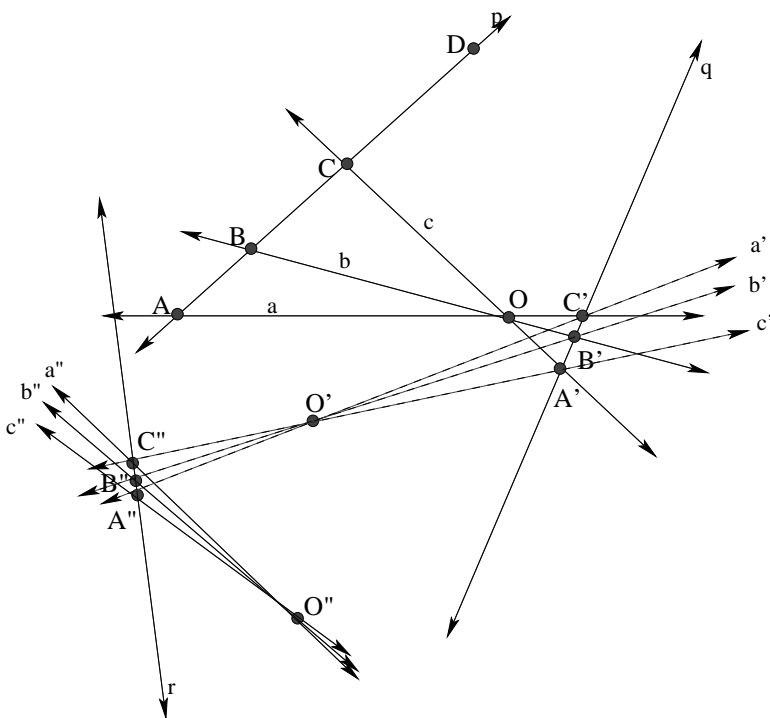
11. The frequency ratio $3 : 4 : 5$ is also equivalent to the ratio $\frac{3}{2} : \frac{15}{8} : \frac{9}{8}$, which gives the chord G, B, D called the dominant of the major triad of the example above. Show $H(OG, DB)$ where $OG = (\frac{2}{3})OC$, $OB = (\frac{8}{15})OC$, and $OD = (\frac{8}{9})OC$.

12. Answer the following questions based on the following diagram:



- (a) Find D , the harmonic conjugate of C with respect to A and B .
- (b) Pick a point E not on \overleftrightarrow{AB} and construct an elementary correspondence between the points A, B, C, D and a pencil of lines with center E .
- (c) Find a line p' distinct from $p = \overleftrightarrow{AB}$ and extend the elementary correspondence you constructed in part (b) to a perspectivity between A, B, C, D and corresponding points on p' .
- (d) Extend this perspectivity to a projectivity $ABC \wedge CDA$.

13. Given the following projectivity:



- (a) Identify each elementary correspondence in this projectivity.
- (b) Find the image of D under this projectivity.