

Due: Friday, April 16th

Given that A is the matrix of an isometry, let X and Y be points in \mathbb{E} . Suppose that $AX = X'$ and $AY = Y'$. Since A is an isometry, $d(X, Y) = d(X', Y')$. Complete the missing steps in the proof of **Proposition 3.7** by proving that if:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}, X' = \begin{bmatrix} x'_1 \\ x'_2 \\ 1 \end{bmatrix}, Y = \begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix}, \text{ and } y = \begin{bmatrix} y'_1 \\ y'_2 \\ 1 \end{bmatrix}, \text{ then:}$$

$$\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} = \sqrt{(x'_1 - y'_1)^2 + (x'_2 - y'_2)^2}$$

$$= \sqrt{(a_{11}^2 + a_{21}^2)(x_1 - y_1)^2 + 2(a_{11}a_{12} + a_{21}a_{22})(x_1 - y_1)(x_2 - y_2) + (a_{12}^2 + a_{22}^2)(x_2 - y_2)^2}$$