

This file is based on the answer key to a Math 310 Practice Exam.

1. Use truth tables to determine whether or not the following pairs of statements are logically equivalent.

(a)  $[(p \wedge q) \rightarrow r]$  and  $(p \rightarrow r) \wedge (q \rightarrow r)$

| $p$ | $q$ | $r$ | $p \wedge q$ | $(p \wedge q) \rightarrow r$ |
|-----|-----|-----|--------------|------------------------------|
| $T$ | $T$ | $T$ | $T$          | $T$                          |
| $T$ | $T$ | $F$ | $T$          | $F$                          |
| $T$ | $F$ | $T$ | $F$          | $T$                          |
| $T$ | $F$ | $F$ | $F$          | $T$                          |
| $F$ | $T$ | $T$ | $F$          | $T$                          |
| $F$ | $T$ | $F$ | $F$          | $T$                          |
| $F$ | $F$ | $T$ | $F$          | $T$                          |
| $F$ | $F$ | $F$ | $F$          | $T$                          |

| $p$ | $q$ | $r$ | $p \rightarrow r$ | $q \rightarrow r$ | $(p \rightarrow r) \wedge (q \rightarrow r)$ |
|-----|-----|-----|-------------------|-------------------|--|
| $T$ | $T$ | $T$ | $T$               | $T$               | $T$  |
| $T$ | $T$ | $F$ | $F$               | $F$               | $F$  |
| $T$ | $F$ | $T$ | $T$               | $T$               | $T$  |
| $T$ | $F$ | $F$ | $F$               | $T$               | $F$  |
| $F$ | $T$ | $T$ | $T$               | $T$               | $T$  |
| $F$ | $T$ | $F$ | $T$               | $F$               | $F$  |
| $F$ | $F$ | $T$ | $T$               | $T$               | $T$  |
| $F$ | $F$ | $F$ | $T$               | $T$               | $T$  |

Since the last columns of these truth tables are not identical, these two propositions are not logically equivalent.

(b)  $p \wedge (q \vee r)$  and  $(p \wedge q) \vee (p \wedge r)$

| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge (q \vee r)$ |
|-----|-----|-----|------------|-----------------------|
| $T$ | $T$ | $T$ | $T$        | $T$                   |
| $T$ | $T$ | $F$ | $T$        | $T$                   |
| $T$ | $F$ | $T$ | $T$        | $T$                   |
| $T$ | $F$ | $F$ | $F$        | $F$                   |
| $F$ | $T$ | $T$ | $T$        | $F$                   |
| $F$ | $T$ | $F$ | $T$        | $F$                   |
| $F$ | $F$ | $T$ | $T$        | $F$                   |
| $F$ | $F$ | $F$ | $F$        | $F$                   |

| $p$ | $q$ | $r$ | $p \wedge q$ | $p \wedge r$ | $(p \wedge q) \vee (p \wedge r)$ |
|-----|-----|-----|--------------|--------------|----------------------------------|
| $T$ | $T$ | $T$ | $T$          | $T$          | $T$                              |
| $T$ | $T$ | $F$ | $T$          | $F$          | $T$                              |
| $T$ | $F$ | $T$ | $F$          | $T$          | $T$                              |
| $T$ | $F$ | $F$ | $F$          | $F$          | $F$                              |
| $F$ | $T$ | $T$ | $F$          | $F$          | $F$                              |
| $F$ | $T$ | $F$ | $F$          | $F$          | $F$                              |
| $F$ | $F$ | $T$ | $F$          | $F$          | $F$                              |
| $F$ | $F$ | $F$ | $F$          | $F$          | $F$                              |

Since the last columns of these truth tables are identical, these two propositions are logically equivalent.