Math 323
Exam 1 - Practice Problems

1. For each of the following curves, first find an equation in x and y whose graph contains the points on the curve. Then sketch the graph of C, indicating its orientation.

   (a) $x = t^2 + 1$ and $y = 2t^2 - 1$
   (b) $x = t^2$ and $y = t^3$
   (c) $x = e^t$, $y = 4e^{2t}$
   (d) $x = 5\cos t$, $y = 3\sin t$
   (e) $x = \cos 2t$, $y = \sin t$

2. Given the parametric curve: $x = 2t^2 + 1$, $y = 3t^3 + 2$

   (a) Find the equation of the tangent line to this parametric curve corresponding when $t = 1$.

   (b) Compute $\frac{d^2y}{dx^2}$ and use this to determine the concavity of the curve when $t = 1$.

3. (a) Graph the parametric curve given by $x = \cos^3 t$, $y = \sin^3 t$.

   (b) Prove that the graph of this parametric curve satisfies the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.

4. Find the arc length of the curve given by:

   (a) $x = 2t$, $y = \frac{2}{3}t^{\frac{3}{2}}$, $5 \leq t \leq 12$
   (b) $x = \sin t - \cos t$, $y = \sin t + \cos t$, $\frac{\pi}{4} \leq t \leq \frac{\pi}{2}$

5. Find the surface area of the solid obtained by revolving the parameterized curve given by revolving $x = t^3$, $y = 2t + 3$ for $0 \leq t \leq 1$ about the y-axis.

6. Express the following polar equations in rectangular coordinates:

   (a) $r = -5\cos\theta$
   (b) $r = \sin(2\theta)$

7. Express the following rectangular equations in polar coordinates:

   (a) $xy = 1$
   (b) $x^2 - y^2 = 1$

8. Find the equation for a circle with center $(0, -4)$ and passing through the origin in both rectangular and polar coordinates.

9. Graph each of the following polar equations:

   (a) $r = 1 - \sin\theta$
   (b) $r = 4 + 2\cos\theta$
   (c) $r = 3\cos(3\theta)$
   (d) $r = 2\sin(5\theta)$
   (e) $r = 3\theta$

10. Find the area of each of the following polar regions:

    (a) the region bounded by the polar graph $r = 1 + \cos\theta$
    (b) the region bounded by one loop of the curve $r = 2\sin(5\theta)$
    (c) the region inside $r = 3 + 2\sin\theta$ and outside $r = 4$
    (d) the region inside both $r = 2\cos\theta$ and $r = 2\sin\theta$

11. Find the arc length of the polar curve $r = 2 - 2\sin\theta$

12. Given the vectors $\vec{a} = \langle 3, -2\rangle$ and $\vec{b} = \langle 2, 1\rangle$, find:

    (a) $\vec{a} + \vec{b}$

    (b) $2\vec{a} - 3\vec{b}$

    (c) $\|3\vec{a} - \vec{b}\|$

    (d) a unit vector in the direction of $\vec{a} - \vec{b}$

    (e) a vector with magnitude 6 in the direction of $2\vec{a} + \vec{b}$

13. Suppose that the thrust of an airplane's engine produces a speed on 400mph in still air and wind velocity is given by $\langle -20, 30\rangle$. Find the direction the plane should head in order to fly due north. Also find the speed at which the plane will travel this course.

14. The water from a fire hose exerts a force of 200 lbs on the person holding the hose. The nozzle of the hose weighs 20 lbs. Find the force required to hold the hose horizontal and find the angle to the horizontal that this force must be applied.

15. Given $P(-1, 0, 2)$ and $Q(1, 2, -4)$

    (a) Plot P and Q.

    (b) Find $d(P, Q)$.

    (c) Find the midpoint of the line segment between P and Q.

    (d) Find the equation for the sphere centered at P and passing through Q.

16. Given the vectors $\vec{v} = 2i - j + 3k$ and $\vec{w} = 4i + 3j - k$

    (a) Find $\vec{v} + \vec{w}$

    (b) Find $2\vec{v} - 3\vec{w}$

    (c) Find $\|\vec{w} - \vec{v}\|$

    (d) Find a unit vector in the same direction as $\vec{v}$.

    (e) Find a vector with magnitude 5 in the opposite direction as $\vec{w}$.

17. Find the force needed to keep a helicopter stationary if the helicopter weighs 1000 lbs and a northeasterly wind exerts a force of 150 lbs on the helicopter.