Math 323
Exam 3 Review Sheet

Section 16.1 Functions of Several Variables

• Understand the definition of a function of several variables and be able to find the domain and range of a given function or several variables and express is algebraically.
• Be able to sketch the graph of the domain of a function of two or three variables.
• Be able to represent the graph of a function of two variables by understanding, graphing, and labeling level curves of the function as part of a contour plot. Also be able to represent the graph of a function of three variables by understanding, graphing, and labeling level surfaces of the function.

Section 16.2 Limits and Continuity

• Understand the definition of the the limit of a function of several variables at a point in its domain.
• Understand the definition of continuity of a function of several variables at a point in its domain and be able to compute the limit of a function of several variables at a point where it is continuous.
• Be able to determine the set of points at which a given function of several variables in continuous.
• Be able to show that the limit of a function of several variables *does not exist* by showing that the limit attains different values when computed along two different paths.
• Understand continuity of the composition of functions of two or more variables.

Section 16.3 Partial Derivatives

• Understand the formal definition of the partial derivatives of a function of several variables.
• Be able to compute the partial derivatives of a given function of several variables and higher order partials of the function as well.
• Understand the meaning of the partial derivative at a given point as the slope of the tangent line to the trace of the function in a given plane.
• Understand that the mixed partials of a continuous function of several variables are equal to each other.

Section 16.4 Increments and Differentials

• Understand the definition of the increment of a function of several variables and be able to compute the general formula for the increment of a given function.
• Memorize the statement of Theorem 16.14, including the conditions on the component functions ε_1 and ε_2.
• Be able to put write the increment of a given function in the form guaranteed by this theorem, identifying the component functions ε_1 and ε_2.
• Understand the definition of the differential of a function of several variables and be able to use the differential of a function to estimate its either its increment or its value.
• Be able to use increments to find estimates for specific application problems. Also be able to determine the error of your estimate by comparing it to the actual increment.
• Know the definition of a differentiable function of several variables and the connection between continuity and differentiability as stated in Theorems 16.17 and 16.18

Section 16.5 Chain Rules

• Understand the chain rule for compositions of functions of one or more variables.
• Be able to apply the chain rule in order to find derivatives and/or partial derivatives involving the composition of functions of one of more variables.
• Be able to use the chain rule to find the derivative or partial derivatives of functions that are given implicitly.

Section 16.6 Directional Derivatives

• Understand the definition of the derivative of a function of several variables in the direction of a **unit** vector $\vec{u}$.
• Be able to compute the directional derivative of a function in a given direction.
• Understand the definition of the gradient of a function of several variables and be able to use it to compute directional derivatives.
• Understand the geometric meaning of the gradient of a function as the direction of the maximum rate of increase. Also be able to use the gradient to find the magnitude and direction of both the maximum and minimum rates of changes of the function at a given point.
• Be able to use directional derivatives in order to find tangent lines to a curve $\mathcal{C}$.

Section 16.7 Tangent planes and Normal Lines

• Understand how to use the gradient of a function to find the equation of the tangent plane to a function of several variables at a given point for functions given both implicitly and explicitly.
• Be able to use the tangent plane to a function in order to approximate the value of a function at a "nearby" point.
• Be able to find the parametric equation of the normal line to a point on the graph of a function of several variables.
• Understand that the gradient vector at a point in the graph of a function of several variables is orthogonal to the level curve (or surface) of the function through that point.

Section 16.8 Extrema of Functions of Several Variables

• Understand critical points and local extrema of functions of several variables.
• Be able to find the critical points of a function of several variables.
• Be able to classify the critical points of a function of several variables using the Discriminant.
• Understand the definition of a saddle point of a function of several variables.
• Understand that a continuous function of several variables has absolute extrema on any closed region in its domain. Be able to find the absolute extrema of a given function by testing its critical points and its boundary points.

Section 16.9 Lagrange Multipliers

• Understand the statement of La Grange's Theorem
• Be able to use La Grange's Theorem to find the extrema of a function of several variables subject to a single constraint.