Math 323
Exam 5 **Name:**______________________________

Instructions: This exam is a "Take Home" exam. You will have until 4:00pm on Wednesday, May 4th to complete this exam. You MAY NOT consult with classmates (or anyone else for that matter) on this exam. The credit given on each problem will be proportional to the amount of correct work shown. Answers without supporting work will receive little credit.

1. (8 points) Find equations that represent the flow lines of the vector field $\vec{F} = \langle -y, xe^y \rangle$

   [You do not need to solve explicitly for y in terms of x.]

2. (8 points) Find the mass of a spring in the shape of $\mathcal{C} = \begin{cases} x = \cos(2t) \\ y = \sin(2t) \\ z = t \end{cases}$ for $0 \leq t \leq \pi$ if its density is given by $\delta(x, y, z) = x^2$.

3. (8 points) Evaluate $\displaystyle\int_{\mathcal{C}} \vec{F} \cdot dr$ if $\vec{F}(x, y, z) = \langle yz + ze^{xz}, xz + z\cos(yz), xy + y\cos(yz) + xe^{xz} \rangle$ and $\mathcal{C}$ is the line segment from $(2, 1, 0)$ to $(-2, 0, 1)$

4. (8 points) Use Green's Theorem to evaluate $\displaystyle\oint_{\mathcal{C}} \left(y\sec^2 x - 2y^3\right) dx + \left(\tan x - 4y^2\right) dy$, where $\mathcal{C}$ is the boundary of the region enclosed by $x = 4 - y^2$ and $x = 0$.

5. (9 points) Use a line integral to calculate the area bounded by $y = x^2$ and $y = 2x$. Then verify your answer by computing the area using a double integral in rectangular coordinates (You must use both specified methods to receive credit on this question).

6. (10 points) Find the mass of surface S with density function $\delta(x, y, z) = e^{\sqrt{x^2+y^2+z^2}}$ if S is the hemisphere $z = \sqrt{4 - x^2 - y^2}$.

7. (12 points) Evaluate the flux integral $\iint_S \vec{F} \cdot \vec{n}\, dS$,
where $\vec{F}(x, y, z) = \langle xy, y^2, z \rangle$, $\vec{n}$ is the outward normal, and S is the boundary of the unit cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.

8. (12 points) Verify Stokes' Theorem for the vector field $\vec{F} = \langle 2x - y, yz^2, y^2z \rangle$ and the surface S given by $z = \sqrt{4 - x^2 - y^2}$ for $z \geq 0$ with upward normal vectors and $\mathcal{C}$ the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of S in the xy-plane by computing both $\oint_{\mathcal{C}} \vec{F} \cdot dr$ and $\iint_S curl(\vec{F}) \cdot \vec{n}\, dS$.