Math 323

Parametric Curves

Definitions:

• A **plane curve** is a set $\mathcal{C}$ of ordered pairs $(f(t), g(t))$, where f and g are continuous functions on an interval I. We will usually refer to $\mathcal{C}$ as a **curve**.

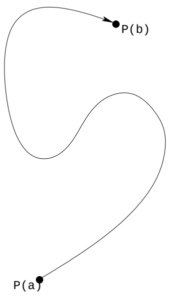
• The **graph** of a curve $\mathcal{C}$ is the set of all points in the plane with coordinates given by $P(t) = (f(t), g(t))$ for some $t \in I$. We often use the term *curve* to refer to both the original curve and its graph. However, we must be careful when doing this, since different curves may have the same graph.

• When I is an interval $[a, b]$, we call $P(a)$ and $P(b)$ the **endpoints** of the curve.

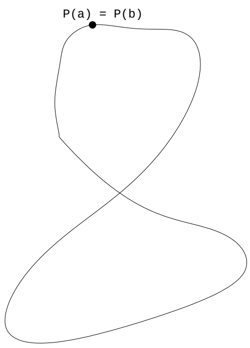
• If $P(a) = P(b)$, then we say that $\mathcal{C}$ is a **closed curve**.

• If a curve $\mathcal{C}$ is closed *and* does not intersect itself at any other point (besides the endpoint) then we say that $\mathcal{C}$ is a **simple closed curve**.

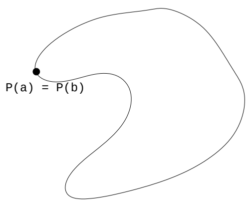
Examples:



a Curve



a Closed Curve



a Simple Closed Curve

• Given a curve $\mathcal{C}$ consisting of all ordered pairs $(f(t), g(t))$, where f and g are continuous functions on an interval I, we say that the equations:

$$C = \begin{cases} x = f(t) & \text{for } t \in I \\ y = g(t) \end{cases}$$

are **parametric equations** for $\mathcal{C}$ with **parameter** t.

• We will call $\mathcal{C}$ a **parameterized curve**, and we will call the equations given above a **parameterization** for $\mathcal{C}$.

Examples: Sketch the graph of each of the following parametric curves.

$$C_1 = \begin{cases} x = t + 1 & \text{for } t \in [-1, 3] \\ y = t^2 \end{cases}$$

$$C_2 = \begin{cases} x = 2\sin t & \text{for } t \in [0, 2\pi] \\ y = 2\cos t \end{cases}$$

$$C_3 = \begin{cases} x = t^2 - 4 & \text{for } t \in \mathbb{R} \\ y = t^2 \end{cases}$$

$$C_4 = \begin{cases} x = e^t & \text{for } t \in \mathbb{R} \\ y = e^t + 4 \end{cases}$$

Note: There are many beautiful and surprising plane graphs that can be described using parametric curves. For those of you who know how to use Maple, try using the standard plot commands given below to generate some interesting parametric curves. Then play with the values of the parameters A and B and see what impact this has on the shape of the curve.

[An Epicycloid] $A := .21; B := .10;$

```
plot([(A+B)*cos(t)-B*cos((A/B+1)*t), (A+B)*sin(t)-B*sin((A/B+1)*t), t = 0 .. 20*Pi]);
```

[A Lissajous curve] $A := .5; B := .4;$

```
plot([A*sin(A*t*Pi), B*sin(B*t*Pi), t = 0 .. 20*Pi]);
```

[An epitrochoid] $A := .55; B := .15; C := .35;$

```
plot([(A+B)*cos(t)-C*cos((A/B+1)*t), (A+B)*sin(t)-C*sin((A/B+1)*t), t = 0 .. 20*Pi]);
```

[A hypotrochoid] $A := .5; B := .1; C := .5;$

```
plot([(A-B)*cos(t)+C*cos((A/B-1)*t), (A-B)*sin(t)-C*sin((A/B-1)*t), t = 0 .. 20*Pi]);
```